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Positioning Empty Containers among Ports Considering Leasing and Purchasing Options Using Robust Optimization

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Abstract

In the competitive shipping industry, container transportation plays a major role. Marine transportation has significantly grown over the past decades, thereby affecting worldwide Trade balances. Empty Container Repositioning (ECR) has always been a serious challenge for carriers and liners. The goal of this study is to propose a model for container carriers and liner companies to improve container transportation between ports and reduce imbalances. This paper presents an optimization model for positioning empty containers among ports, accounting for leasing and purchasing costs, using robust programming. The proposed model's objective is to minimize the relevant costs container carriers often face in the business environment, including transportation, inventory, leasing and purchasing, and handling costs. Results from the developed robust optimization model showed an approximately 70% increase in total cost in the worst-case scenario. Container leasing capacity and vessel size have been modeled using mixed-integer programming and robust optimization in the proposed model.

Keywords: Container transportation, Empty container positioning, Marine transportation, Mixed integer programming, Robust optimization.

1 | Introduction

Since the 1970s, maritime transportation has increased sharply due to rising global trade. In 2017, approximately 17.1% of world seaborne trade was transported in containers.

In the global shipping industry, effective container positioning is paramount for minimizing operational costs and enhancing logistical efficiency [1].

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Empty container positioning is a critical aspect of maritime logistics, directly influencing port efficiency and operational costs, which involves strategically distributing empty containers across ports based on fluctuating demand, shipping schedules, and economic factors. Ports experience variable requirements depending on seasonality, trade routes, and the influx of import/export activities.

When the number of import containers exceeds the number of export containers, some ports have a surplus of empty containers, while others have a deficit. When ports have a deficit of empty containers, shipping companies must lease or purchase them; when ports have a surplus, the empty containers are stored in depots. Strategically, positioning empty containers is therefore one of the most effective ways to address container imbalance [2].

One pivotal aspect of this optimization model is the decision about leasing versus purchasing containers. Leasing provides flexibility, allowing companies to adapt to rapidly changing market conditions without incurring the high costs of ownership (used to cover short-term empty container shortages). Purchasing, on the other hand, may offer long-term savings but ties up capital and assets, increasing financial risk in an unstable market (depending on the shipping company's strategy).

According to Kuzmicz and Pesch [3], containers spend about 56% of their 10-15-year lifespan stacked at depots or being repositioned empty. The key decision is when and how many empty containers are repositioned to a destination port [1].

Models used to simulate this industry's environment must incorporate constraints such as port storage capacity, maintenance costs, container turnaround times, etc. By systematically analyzing these factors, firms can develop a comprehensive strategy for empty container positioning that maximizes availability while minimizing costs.

The integration of robust optimization models into container positioning strategies offers an advanced approach to managing the complexities surrounding leasing versus purchasing. This method not only clarifies the decision-making process but also ensures that shipping companies remain agile and competitive in a dynamic maritime market, ultimately leading to enhanced resource allocation and improved service delivery across global ports.

The core of a robust optimization model is its ability to accommodate worst-case scenarios for uncertain parameters, such as fluctuations in demand for container leasing or purchasing, variable transportation costs, and changing maritime policies. The Bertsimas and Sim budget robust framework provides a structured approach to determine the portfolio of container acquisition strategies, whether leasing or purchasing, under limited budgets, while accounting for uncertainty.

In this paper, we consider the issue discussed by Moon et al. [2], namely, ocean-empty-container positioning planning. We introduce a robust model based on the Moon et al. [2] model and consider uncertainties in costs and rates associated with repositioning empty containers. In addition, to meet the demand, we also consider short-term leasing and purchasing factors. In long-term leasing, we can treat the leased container as owned, assuming the lease is for a long period. However, in short-term leasing, the lease duration is short (in several periods), and container handling is required when the container is returned. The objective of this study is to minimize the total relevant costs, including transportation, handling, inventory holding, leasing, and purchasing costs. The remaining content of the paper is organized as follows: in Section 2, we introduce the robust mathematical model based on Moon et al. [2]. In Section 3, we present the computational results for both the deterministic and robust models and, in the end, compare the two models. In Section 4, a sensitivity analysis of the effects of parameters related to leasing and returning leased containers, as well as to owned vessels' capacity, on total network costs has been presented. Finally, in Section 5, we present the conclusion and outline future research directions related to positioning empty containers.

2 | Literature Review

Regarding the importance of positioning empty containers for ocean carriers and port authorities, numerous studies have been conducted over the past decade.

Wang et al. [4] examined the problem of locating shipping hub containers and positioning empty containers alongside them, accounting for collapsible containers. To this end, they present a mixed-integer programming model and subsequently address the pricing of empty container rental within it. Abdelshafie et al. [5] introduced a modeling framework based on agent-based modeling for Empty Container Repositioning (ECR). Song et al. [6] developed three optimization models based on three strategies in ECR among ports. The three Considered strategies are 1) full allocation, 2) no allocation, and 3) near allocation bound. The chain calculation method has been used in this paper. Bakir et al. [7] introduced a robust model that considers stochastic demand for positioning empty containers among ports, to minimize costs related to the repositioning process, and the methodology used in the paper is budget-robust optimization. Song et al. [6] considered ECR with stochastic demand and stochastic container supply across multiple ports. The two-stage model used in the paper has been calculated using the particle swarm optimization algorithm and the Markov chain. Feng et al. [8] introduced a multi-stage decision-making model for empty container positioning to minimize lorry costs and idle time. By treating trade imbalance as the primary driver of container repositioning, Gencer and Demir [9] developed deterministic and stochastic models to minimize the costs associated with container repositioning. The deterministic model has been solved using mixed-integer programming, and the stochastic model, considering uncertain demand, has been solved using scenario-based stochastic programming. Lu et al. [10] introduced a stochastic dynamic programming model with uncertain container demand to account for both repositioning empty containers and pricing factors, as well as their coordination, for a shipping company operating in a two-container depot system. Zhang and Facanha [11] conducted a case study on repositioning empty containers for one of the top shipping carriers in the world in the transpacific corridor, then, based on that, developed a model considering uncertain demand and supply at the strategic level. In the end, he offered three strategic propositions to enhance progress and reduce related costs. Zhang et al. [12] introduced a stochastic model to reduce costs associated with positioning empty containers, accounting for uncertain demand and lost sales. Di Francesco [13] studied positioning empty containers among ports considering uncertainty in port disruption and modeled the subject using multi-scenario stochastic programming. Dong et al. [14], after giving a literature review, intended to analyze two strategies: 1) origin-destination, and 2) dynamic rules based on conditions) for positioning empty containers and comparing them.

Moon et al. [15] calculated and compared the total costs of positioning containers among ports by considering two scenarios: 1) using foldable containers, and 2) using standard containers. The model heuristic algorithm has been used. Meng and wang [16] in their work involve designing a network model considering the flow passage through hub ports, multiple regular ports, and the positioning of empty containers. In the first step, pairs of ports served by a shipping company are considered, and a mixed-integer linear programming model is then defined. Real operational data from Pacific-Asia-Europe shipping lines are applied to this model, and results are obtained based on real-world data.

Additionally, to demonstrate the net capital preservation of the model across the entire network, the model was finally assessed without considering the allocation of empty containers. Moon et al. [2] developed an optimization model to account for costs associated with positioning empty containers, including fixed and variable container shipping costs, Terminal Handling Costs (THC) at ports, and other related expenses, as well as the costs of renting and purchasing empty containers. In the end, it utilized heuristic, genetic, and hybrid algorithms to solve the problem.

3 | Problem Definition

As our contribution is to develop a robust optimization model based on Moon et al. [2], we adopt the same problem definition as demonstrated in Moon et al. [2]. We consider a scenario for describing a shipping company's activities. The shipping company receives an order at a port from a customer and, after that, attempts to send an empty container to the customer. The customer's cargo will then be loaded into the received empty containers. After staffing, the containers are completed, and the full containers are sent to the nearest port container terminal, loaded onto a vessel, and carried to the destination port. At the destination port, after unloading the full delivered containers, they are then transported to the receiver. By the time the receiver completes stripping, the containers are delivered back empty to the depot. After inspection and maintenance, those empty containers are ready for another trip. When full containers are received from another port, they are imported; if they are transported to other ports, they are exported. If the number of imported containers is either fewer or more than the number of exported containers, there will be an imbalance. If the number of exported containers exceeds the number imported at a port, there will be a shortage of empty containers.

On the other hand, for ports with more imports than exports, there will be a surplus of empty containers. At surplus ports, surplus empty containers are stored in empty container depots, and a certain cost is incurred for holding them, in addition to the fact that these valuable assets will remain idle for some time. At the deficit ports, there is no choice but to lease or purchase empty containers to meet demand. The shipping company must find a solution to reduce those costs. One solution could be "empty container positioning," which is transporting empty containers from surplus to deficit ports. By positioning empty containers at ports, costs related to leasing, purchasing, and inventory are reduced, but there will be transportation costs. Moreover, due to demand fluctuations, a port might have a surplus at some times and a deficit at others. Therefore, the question for the shipping company is: How many empty containers of different types, and when and where are they supposed to be positioned? We therefore aim to propose a mathematical model to answer this question. The rest of the problem definition can be seen in [2].

4 | Modeling

4.1 | Assumptions

Assumptions of the mathematical model are as follows [2]:

I. Demand must be satisfied at each period, and no backlog is permitted.

II. The total capacity of both owned and rented vessels is enough to transport full containers for a certain period.

III. The routing of the vessels has not been considered.

IV. Full containers will be emptied after arriving at the destination port within one period.

V. Purchasing empty containers has no limitations.

4.2 | Budget Robust Model

Based on both the deterministic [2] and the budget-robust counterpart model [17], we now attempt to develop and introduce the robust model.

Robust parameters

Γ_{RV1}	Uncertainty budget of parameter RV1.
Γ_{RV2}	Uncertainty budget of parameter RV2.
Γ_{TV1}	Uncertainty budget of parameter TV1.
Γ_{TV2}	Uncertainty budget of parameter TV2.
Γ_{LV}	Uncertainty budget of parameter LV.
Γ_{PC}	Uncertainty budget of parameter PC.
Γ_{D}	Uncertainty budget of parameter D, which is a variable between [-1,1].
Ψ_{D}	Forecasted fluctuation for parameter D.

Robust decision variables

P_{jvi}^{RV1}	Perturbation variable of parameter RV1.
U^{RV1}	Perturbation variable of parameter RV1.
P_{jvi}^{RV2}	Perturbation variable of parameter RV2.
U^{RV2}	Perturbation variable of parameter RV2.
P_{ivj}^{TV1}	Perturbation variable of parameter TV1.
U^{TV1}	Perturbation variable of parameter TV1.
P_{ivj}^{TV2}	Perturbation variable of parameter TV2.
U^{TV2}	Perturbation variable of parameter TV2.
P_{kv}^{LV}	Perturbation variable of parameter LV.
U^{LV}	Perturbation variable of parameter LV.
P_{iv}^{PC}	Perturbation variable of parameter PC.
U^{PC}	Perturbation variable of parameter PC.

Objective function

$$Z' = \sum_{i}^{P}\sum_{j,j\neq i}^{P}\sum_{t>Ti_{ij}}^{T}[(RF_{ji}^{1}\times S_{ji,t-TI_{ji}}^{1} + \sum_{v}^{V} RV_{jiv}^{1}\times F_{jiv,t-TI_{ji}}^{1}) + (RF_{ji}^{2}\times S_{ji,t-TI_{ij}}^{2} + \sum_{v}^{V} RV_{jiv}^{2}\times F_{jiv,t-TI_{ij}}^{2})] +$$

$$\sum_{i}^{P}\sum_{j,j\neq i}^{P}\sum_{t=1}^{T}[(TF_{ij}^{1}\times S_{ijt}^{1} + \sum_{v}^{V} TV_{ijv}^{1}\times F_{ijvt}^{1}) + (TF_{ij}^{2}\times S_{ijt}^{2} + \sum_{v}^{V} TV_{ijv}^{2}\times F_{ijvt}^{2})] +$$

$$\sum_{i}^{P}\sum_{v}^{V}\sum_{t=1}^{T}[H_{iv}\times(I_{ivt}+\sum_{k}^{P}LI_{ivt}^{K})]+\sum_{i}^{P}\sum_{v}^{V}\sum_{t=1}^{T}(LF_{iv}\times U_{ivt})+\sum_{k}^{P}\sum_{i}^{P}\sum_{v}^{V}\sum_{t=1}^{T}(LV_{kv}\times LI_{ivt}^{k})+$$

$$\sum_{k}^{P}\sum_{i}^{P}\sum_{j,j\neq i}^{P}\sum_{v}^{V}\sum_{t=1}^{T}(LV_{kv}\times(X_{ijvt}^{elk}+X_{ijvt}^{flk})\times TI_{ij})+\sum_{i}^{P}\sum_{v}^{V}\sum_{t=1}^{T}(PC_{iv}\times P_{ivt})+(\Gamma_{RV1}\times U^{RV1})+\sum_{j}^{P}\sum_{v}^{V}\sum_{i,i\neq j}^{P}P_{jvi}^{RV1}$$

$$+(\Gamma_{RV2}\times U^{RV2})+\sum_{j}^{P}\sum_{v}^{V}\sum_{i,i\neq j}^{P}P_{jvi}^{RV2}+(\Gamma_{TV1}\times U^{TV1})+\sum_{i,i\neq j}^{P}\sum_{v}^{V}\sum_{j}^{P}P_{ivj}^{TV1}+(\Gamma_{TV2}\times U^{TV2})+\sum_{i,i\neq j}^{P}\sum_{v}^{V}\sum_{j}^{P}P_{ivj}^{TV2}$$

$$+(\Gamma_{LV}\times U^{LV})+\sum_{k}^{P}\sum_{v}^{V}P_{kv}^{LV}+(\Gamma_{PC}\times U^{PC})+\sum_{i}^{P}\sum_{v}^{V}P_{iv}^{PC}.$$

Constraints

$$X_{ijvt}^{f}+\sum_{k}^{P}X_{ijvt}^{flk}=D_{ijvt}+(\Gamma_{D}\times\Psi_{D}),\ \text{for all } i,j,v,t,,i\neq j, \tag{10}$$

$$U_{RV1}+P_{jvi}^{RV1}\geq RV_{jvi}^{1}\times F_{jiv,t-TI_{(j,i)}}^{1},\ \text{for all } i,j,v,t,\ i\neq j,t>TI_{(j,i)}, \tag{11}$$

$$U_{RV2}+P_{jvi}^{RV2}\geq RV_{jvi}^{2}\times F_{jiv,t-TI_{(j,i)}}^{2},\ \text{for all } i,j,v,t,\ i\neq j,t>TI_{(j,i)}, \tag{12}$$

$$U_{TV1}+P_{ivj}^{TV1}\geq TV_{ivj}^{1}\times F_{ijvt}^{1},\ \text{for all } i,j,v,t,\ i\neq j, \tag{13}$$

$$U_{TV2}+P_{ivj}^{TV2}\geq TV_{ivj}^{2}\times F_{ijvt}^{2},\ \text{for all } ,j,v,t,\ i\neq j, \tag{14}$$

$$U_{LV}+P_{kv}^{LV}\geq LV_{kv}\times LI_{ivt}^{k}+LV_{kv}\times(X_{ijvt}^{elk}+X_{ijvt}^{flk})\times TI_{ij}\ \text{for all } i,j,k,v,t,\ i\neq j, \tag{15}$$

$$U_{PC}+P_{iv}^{PC}\geq PC_{iv}\times P_{ivt},\ \text{for all } i,v,t, \tag{16}$$

$$U_{PC},P_{iv}^{PC},U_{LV},P_{kv}^{LV},U_{TV2},P_{ivj}^{TV2},U_{TV1},P_{ivj}^{TV1},U_{RV1},P_{jvi}^{RV1},U_{RV2},P_{jvi}^{RV2}\geq 0. \tag{17}$$

Constraint (10) is the replacement for *Constraint (3)* in the deterministic model, serving as the new demand satisfaction constraint in the robust model. *Constraint (11)* sets a lower bound on the perturbation value of parameter RV1, which corresponds to the variable handling cost in ports for containers transported by the carrier's vessels. *Constraint (12),* similar to *Constraint (11)*, does the same for containers transported by rented vessels (parameter RV2). *Constraint (13)* sets a lower bound on the perturbation value of parameter TV1, which is related to the variable transportation cost of containers by owned vessels. At the same time, Constraint (14) does the same for those transported by rented vessels. *Constraint (15)* sets a lower bound for parameter LV (variable container leasing costs). *Constraint (16)* sets a lower bound on parameter PC, the container purchasing cost.

Constraints of the deterministic model, as shown in Moon et al. [2], except for the demand satisfaction constraint, are included in the robust model as well.

5 | Computational Results

After introducing the robust model in Section 2, we now present the results of both models and, in the end, compare them to understand how uncertainty can affect deterministic model results.

5.1 | Parameters

Before attempting to show the model results, we illustrate the value of some parameters given:

Table 1. Given the value of the fixed handling cost.

Par: RV1	Shanghai	Rajaei	Jebel Ali	Rotterdam	Hamburg	S.petersburg	Singapore
Shanghai	0	500000	550000	430000	460000	560000	600000
Rajaei	580000	0	550000	430000	460000	560000	600000
Jebel Ali	580000	500000	0	430000	460000	560000	600000
Rotterdam	580000	500000	550000	0	460000	560000	600000
Hamburg	580000	500000	550000	430000	0	560000	600000
S.petersburg	580000	500000	550000	430000	460000	0	600000
Singapore	580000	500000	550000	430000	460000	560000	0

Table 2. Capacity of each container depot at ports for each type of container.

Par: K	20ft	40ft
Shanghai	1800	1000
Rajaei	1800	1000
Jebel Ali	1800	1000
Rotterdam	1800	1000
Hamburg	1800	1000
S.petersburg	1800	1000
Singapore	1800	1000

Table 3. Voyage time between the origin and destination ports.

Par: TI	Shanghai	Rajaei	Jebel Ali	Rotterdam	Hamburg	S.petersburg	Singapore
Shanghai	0	2	2	2	3	3	1
Rajaei	2	0	1	2	2	2	1
Jebel Ali	2	1	0	2	2	2	1
Rotterdam	2	2	2	0	1	1	2
Hamburg	3	2	2	1	0	1	2
S.petersburg	3	2	2	1	1	0	2
Singapore	1	1	1	2	2	2	0

Table 4. The purchasing cost of each container type at ports.

Par: PC	20ft	40ft
Shanghai	2600	5725
Rajaei	2000	2500
Jebel Ali	2100	2800
Rotterdam	2450	3250
Hamburg	2500	3300
S.petersburg	2400	3000
Singapore	2800	5000

Table 5. Leasing capacity of each container type at ports.

Par: LK	20ft	40ft
Shanghai	600	500
Rajaei	600	500
Jebel Ali	600	500
Rotterdam	600	500
Hamburg	600	500
S.petersburg	600	500
Singapore	600	500

5.2 | Deterministic Model Results

We now demonstrate the results acquired from Moon et al. [2], who introduced a deterministic model:

Table 6. Inventory of owned containers at the end of each period at each port.

Port	Container Type	Time Period	Owned Containers
Shanghai	20ft	1	148
Shanghai	20ft	7	86
Shanghai	40ft	10	18
Rajaei	20ft	7	41
Rajaei	40ft	9	104
Jebel Ali	20ft	1	240
Jebel Ali	40ft	1	164
Jebel Ali	40ft	6	19
Jebel Ali	40ft	9	188
Rotterdam	20ft	1	103
Rotterdam	40ft	1	45
Rotterdam	40ft	4	62
Hamburg	20ft	1	114
Hamburg	20ft	11	213
S.petersburg	20ft	7	312
S.petersburg	40ft	1	45
Singapore	20ft	7	83
Singapore	20ft	8	172
Singapore	40ft	1	204
Singapore	40ft	5	5
Singapore	40ft	9	164

Table 7. Inventory of leased containers at the end of each period at each port that are leased in a different port.

Port	Container Type	Time Period	Leased Port	Leased Containers
Shanghai	20ft	10	S.petersburg	214
Shanghai	40ft	6	Shanghai	11
Shanghai	40ft	10	Shanghai	86
Rotterdam	20ft	5	Shanghai	168
Rotterdam	20ft	5	Rotterdam	110
Rotterdam	20ft	5	Singapore	41
S.petersburg	20ft	10	Shanghai	139
S.petersburg	20ft	10	Singapore	11

Table 8. Number of containers to be transported in multiple periods by rented vessels.

Origin Port	Destination Port	Container Type	Time Period	Containers Transported
Shanghai	Hamburg	40ft	8	153
Shanghai	Singapore	40ft	1	141
Shanghai	Singapore	40ft	9	143
Shanghai	Singapore	40ft	11	143
Shanghai	Singapore	40ft	12	143
Rajaei	Shanghai	40ft	1	952
Rajaei	Shanghai	40ft	2	952
Rajaei	Shanghai	40ft	3	421
Rajaei	Jebel Ali	40ft	11	188
Rajaei	Rotterdam	40ft	3	154

Table 8. Continued.

Origin Port	Destination Port	Container Type	Time Period	Containers Transported
Jebel Ali	S.petersburg	40ft	5	163
Jebel Ali	Singapore	40ft	8	163
Rotterdam	Hamburg	20ft	7	208
Rotterdam	Hamburg	20ft	11	170
Hamburg	Rajaei	20ft	4	144
Hamburg	Rajaei	20ft	5	144

5.3 | Robust Model Results

As mentioned before, to account for uncertainty in the real-world environment, we considered seven parameters: demand; variable handling costs for containers transported by both owned and rented vessels at ports; variable transportation costs for both owned and rented vessels; variable container leasing costs; and container purchasing costs. In this section, using the developed budget-robust model, we demonstrate the effects of perturbations on the model's results.

Table 7. Perturbation value of the container variable leasing cost parameter.

Port	Container Type	Perturbation Value
Shanghai	20ft	131400
Shanghai	40ft	175700
Rajaei	20ft	60000
Rajaei	40ft	87500
Jebel Ali	20ft	60000
Jebel Ali	40ft	87500
Rotterdam	20ft	99600
Rotterdam	40ft	87500
Hamburg	20ft	80000
Hamburg	40ft	87500
S.petersburg	20ft	155400
S.petersburg	40ft	87500
Singapore	20ft	80000
Singapore	40ft	140000

Table 8. Perturbation value of the container purchasing cost parameter.

Port	Container Type	Perturbation Value
Shanghai	20ft	45086600
Shanghai	40ft	82239625
Rajaei	20ft	35566000
Rajaei	40ft	38352500
Jebel Ali	20ft	36947400
Jebel Ali	40ft	39592000
Rotterdam	20ft	42441350
Rotterdam	40ft	46185750
Hamburg	20ft	42955000
Hamburg	40ft	47939100
S.petersburg	20ft	41884800
S.petersburg	40ft	42870000
Singapore	20ft	49350000
Singapore	40ft	71680000

Table 9. perturbation value of variable handling cost at destination ports for containers that are transported by owned vessels.

Destination Port	Container Type	Origin Port	Perturbation Value
Shanghai	40ft	Rajaei	54450
Shanghai	40ft	Jebel Ali	49950
Shanghai	40ft	Hamburg	68850
Shanghai	40ft	Singapore	64350
Rajaei	40ft	Shanghai	428400
Rajaei	40ft	Jebel Ali	84600
Rajaei	40ft	Rotterdam	69300
Rajaei	40ft	S.petersburg	50400
Jebel Ali	40ft	Rotterdam	57150
Jebel Ali	40ft	S.petersburg	73350
Jebel Ali	40ft	Singapore	73350
Rotterdam	20ft	Hamburg	52000
Rotterdam	40ft	Shanghai	47700
Rotterdam	40ft	Jebel Ali	55350
Hamburg	40ft	Rotterdam	51750
S.petersburg	40ft	Jebel Ali	63000
Singapore	40ft	Shanghai	50400
Singapore	40ft	Hamburg	55350
Singapore	40ft	S.petersburg	55350

5.4 | Comparison

Now, by considering results from both deterministic and robust models, we can compare them to see how uncertainty can affect models, just as it does in real-world environments. Such a comparison will provide an accurate proof of the importance of considering uncertainty in programming to keep results as close as possible to the real-world environment.

Table 10. Comparison between the objective functions of both deterministic and robust models.

Difference	Budget Robust Model Obj Function	Deterministic Model Obj Function	
71.13% increase	5005485175	2924853700	Considering demand uncertainty
20.27% increase	3517953325	2924853700	Not considering demand uncertainty

As demonstrated in *Table 13*, we observe a significant increase in the objective function value in the worst-case scenario. To reach the worst case, we attempted to increase the perturbation budget for the uncertain parameters over 50 iterations. At each iteration, the budget-robust method increases the objective function value with the allowable perturbation to uncertain parameters. Still, after a certain amount of increase in the perturbation budget, the objective function stops increasing and becomes stable. By reaching stability in the objective function after an increase, we can conclude that the model has reached its worst-case scenario and won't get any worse.

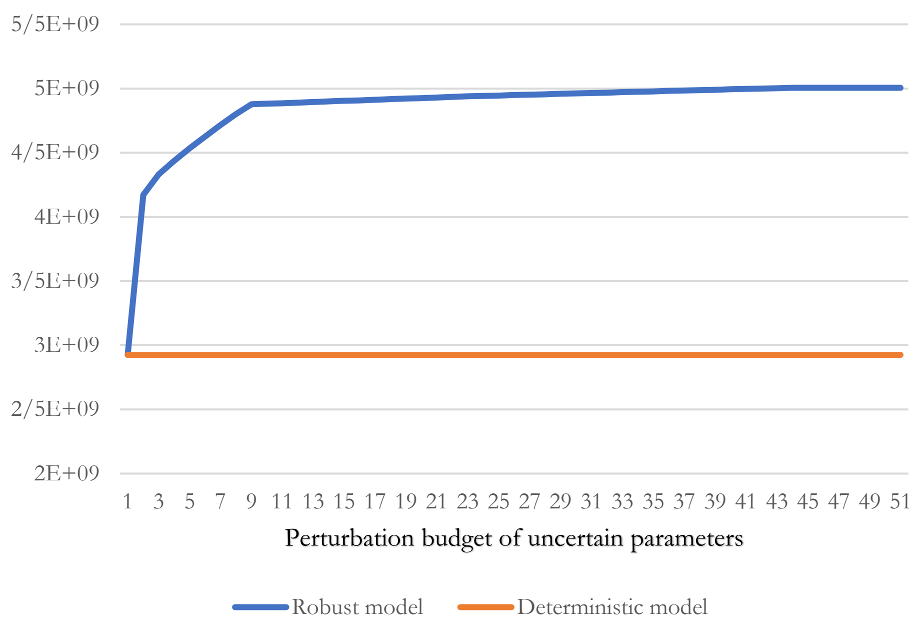

Fig. 1. Comparison between the two models' objective values.

According to *Fig. 2*, after reaching iteration 43 and allocating a perturbation budget of 86 to the uncertain parameters, the model reaches its worst and doesn't improve thereafter.

6 | Sensitivity Analysis

We now attempt to analyze the effects of some parameters in the deterministic model on the objective function and total costs in this section. We consider three parameters.

6.1 | Container Leasing

In the real world, shipping companies face limitations in both leasing and purchasing containers for many reasons, including their long- or short-term strategies, the lack of containers at required ports, and insufficient budgets. Hence, we analyze the effects of both container leasing capacity and the return capacity of leased containers at each port on total costs.

Table 11. Changes in container leasing capacity and the return of leased containers' capacity.

	Leasing Capacity	Returning if Leased Containers Capacity	Obj Function (Total Cost)
1	600	400	2924853700
2	800	600	2907228350
3	1000	800	2890451400
4	1200	1000	2873847000
5	1500	1300	2849289200
6	2000	1800	2808242200
7	2500	2300	2768135150
8	3000	2800	2728735500
9	4000	3800	2652730550
10	5000	4800	2580435475
11	10000	9800	2242153475
12	15000	14800	2154599375
13	30000	29000	2154599375
14	40000	39000	2154599375
15	∞	∞	2154599375

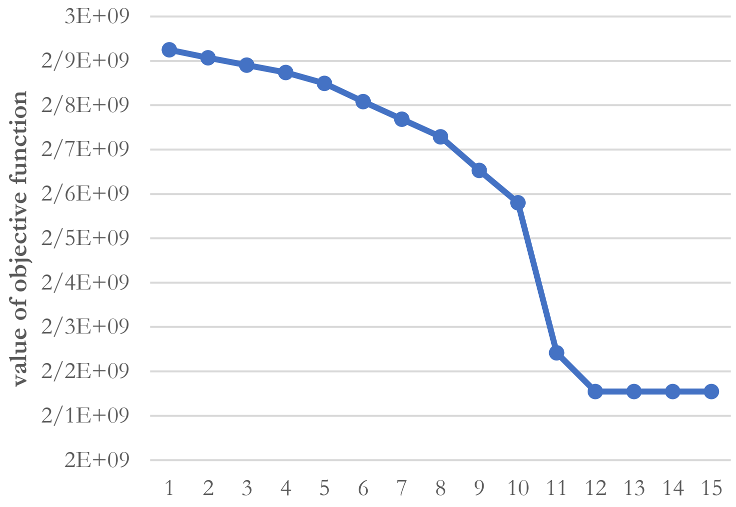

Fig. 1. Effects of the mentioned parameters on the value of the objective function.

Based on *Table 14*, we present *Fig. 3*, which shows the change in the objective function (total cost). As shown, by increasing the capacity for leasing and returning leased containers, total cost decreases substantially, but after level 12, it ceases decreasing. The reason for such behavior is that leasing containers is a temporary solution to the short-term empty container deficit and can impose higher long-term leasing costs than purchasing containers from the beginning. Hence, no matter the leasing and returning capacity, to show the optimal solution, the model will not attempt to lease empty containers and return them beyond a certain quantity.

6.2 | Owned Vessels Capacity

Container vessels are among the most valuable assets for shipping companies; the higher the capacity, the lower the transportation costs. Even Liner shipping companies attempt to form alliances with companies that own vessels with greater capacity. Aside from vessel capacity, total operational fleet capacity is equally important as each vessel's capacity. When a shipping company owns a large fleet of vessels, it can easily assign the right vessel to the right number of containers at origin ports.

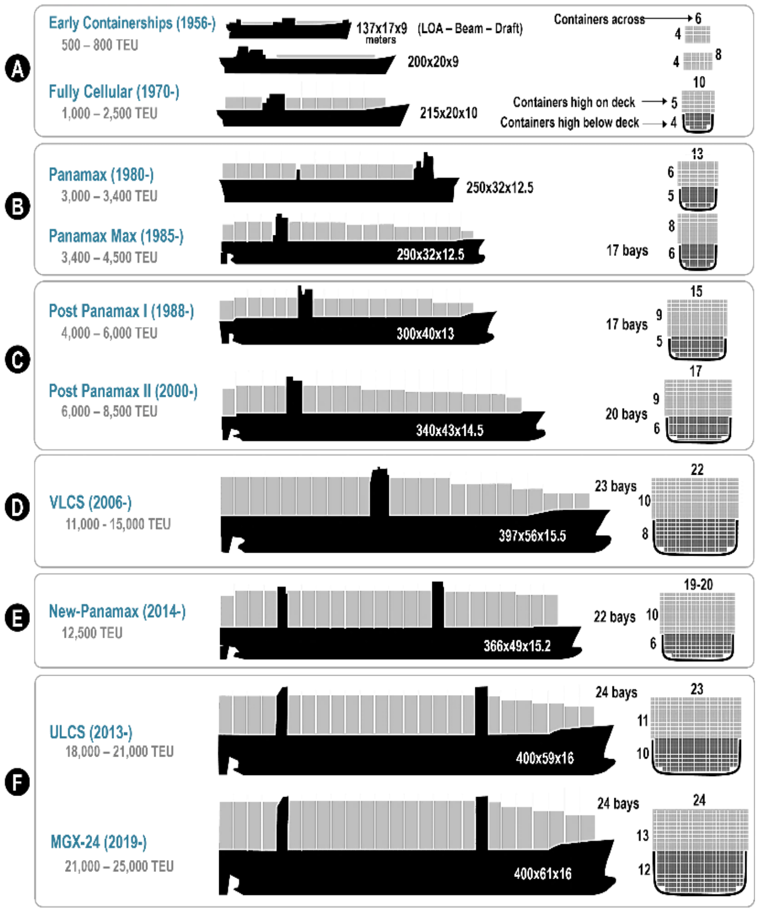

Fig. 2. Types of container vessels.

We aim to evaluate the effects of higher fleet capacity on the total cost of the network.

Table 12. Effects of the value of owned vessels' capacity on the objective function (total cost).

	Capacity of Vessels (TEU)	Obj Function (Total Cost)
1	8000	2924853700
2	9000	2919919900
3	10000	2915974650
4	11000	2912863200
5	12000	2910278000
6	15000	2904653050
7	20000	2900330525
8	25000	2898388875
9	30000	2897374425
10	35000	2897374425
11	∞	2897374425

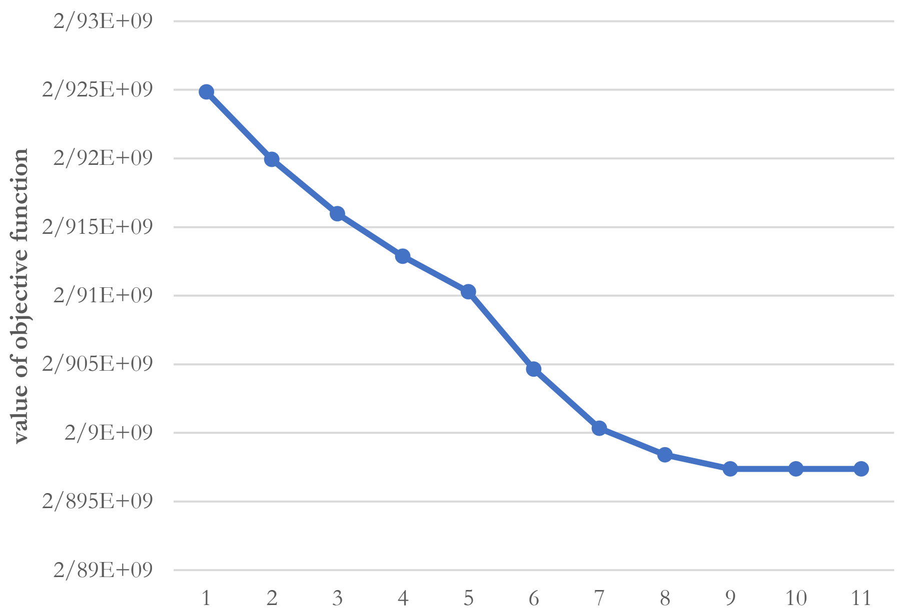

Fig. 3. Effects of the mentioned parameters on the value of the objective function.

A glance at *Fig. 5* shows the effectiveness of vessels with greater capacity, as evidenced by the reduction in total network cost. After level 9, we can see that the objective function ceases decreasing and remains constant. When a vessel's capacity is large enough to meet the demand for transporting both laden and empty containers between origin and destination ports, economies of scale provide higher efficiency and lower transportation costs. Also, with sufficient capacity, there will be no need to rent vessels, so the fixed and variable costs of transportation via rented vessels won't be considered, and the total network cost will decrease as a result.

7 | Conclusion

In this article, we used the deterministic model of Moon et al. [2] as the basis for our developed robust optimization model. A robust method has been used to minimize the effects of uncertainty in the worst-case scenario. Seven parameters, including variable handling costs at ports for both owned and rented vessels, carriable transportation costs for both owned and rented vessels, variable leasing costs for containers, container purchasing costs, and demand, were considered uncertain in our developed robust model. Based on results from both deterministic and robust models, and by understanding the importance of considering uncertainty in mathematical models, our developed robust model produced more realistic results than the deterministic model. Capacity of leasing and returning leased containers plays a vital role in repositioning costs, however, it cannot completely replace purchasing containers if we are eager to achieve the optimal solution (In the real world environment, based on the strategy of each shipping company, a shipping company might choose to rent all of its assets including vessels, containers, depots and, etc. while another company chooses only to purchase assets permanently and some others might choose to both purchase and lease at the same time). Fleet capacity also plays an important role. Shipping companies with larger fleets tend to leverage economies of scale to gain an advantage over the competition and reduce costs effectively.

For future research, considering foldable containers in the model would elevate the value of the introduced model. Using other methods of robust optimization could also be another possible extension. In the end, developing a model with two objective functions to minimize costs and maximize profit simultaneously would definitely help shipping companies achieve optimal solutions.

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Data Availability

The data used in this study are available from the corresponding author upon reasonable request.

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