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Vehicle Routing Problem with Delivery Options and Roaming Delivery Locations

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Abstract


This study examines a variation of the Vehicle Routing Problem (VRP) that incorporates roaming delivery locations and flexible delivery options. In this model, each customer may be served at one of several potential locations, including Shared Delivery Centers (SDC) that provide centralized access to packages. To simultaneously optimize route planning, delivery location selection, and service timing while accounting for shared center limitations, vehicle capacity, and time windows, a mixed-integer linear programming formulation is proposed. The model is evaluated against a baseline scenario in which each customer has a single fixed delivery point. Computational results from a numerical example demonstrate that enabling roaming and flexible delivery can significantly improve efficiency by reducing vehicle usage, maintaining full customer coverage without increasing waiting times, and cutting total costs by more than 40%. These findings highlight practical strategies for optimizing urban distribution and emphasize the operational advantages of offering delivery choices in last-mile logistics.


Keywords: Urban last mile logistics, Vehicle routing problem, Roaming delivery locations, Vehicle routing problem with delivery options, Attended home delivery, Shared delivery locations.

1 | Introduction

The explosive growth of e-commerce and the rapidly changing customer expectations have driven a significant transformation in last-mile logistics in recent years. This shift in consumer behavior was further accelerated by the COVID-19 pandemic, which led to a strong preference for flexible, contactless delivery methods [1], [2]. Today’s customers demand a range of delivery options, including home delivery, parcel lockers, and shared collection points, alongside prompt and dependable service. Consequently, logistics providers now face additional operational challenges, underscoring the need for more flexible, customer-focused distribution strategies.

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The Vehicle Routing Problem (VRP) is a well-known and extensively studied logistics optimization problem that seeks to determine the most effective routes for a fleet of vehicles to serve a set of customers under various constraints [3]. Recent extensions of the VRP have incorporated advanced features such as time windows, heterogeneous fleets, and flexible delivery options. Among these, the concept of roaming delivery, where a customer can be served at multiple potential locations, has gained increasing attention [4]. However, much of the existing research focuses on dynamic routing to private addresses. It often overlooks Shared Delivery Locations (SDLs), such as parcel lockers or common pickup points, when compiling the customer's candidate delivery list. Moreover, these shared facilities are frequently modeled with unlimited capacity, disregarding practical limitations such as congestion, service fairness, and operational constraints. As a result, current approaches may fail to capture both the logistical benefits and the real-world challenges associated with shared delivery infrastructure in last-mile distribution systems.

This study explores a single-depot VRP that permits SDLs and accounts for delivery options at a more detailed level, thereby addressing this gap. As illustrated in *Fig. 1*, the delivery network includes both private home delivery points (circles) and SDLs (pentagons). Each dashed oval region (L1, L2, L3) represents an SDL cluster where customers can choose to receive their packages instead of home delivery. A single central depot (square) dispatches vehicles that visit a mix of home addresses and shared centers depending on the routing decisions. Additionally, each color in the figure corresponds to a unique customer, meaning that a customer may have multiple delivery options (private or shared), all represented with the same color across different nodes. This visual layout illustrates how incorporating roaming and centralized pickup options into routing plans increases delivery flexibility while maintaining service efficiency.

In contrast to conventional models, this study assesses two frameworks: one with fixed delivery locations and another with centralized pickup locations and roaming delivery options. The comparative analysis demonstrates significant efficiency gains enabled by flexible delivery structures, including reduced fleet size and operating costs. This research advances the theoretical development and practical implementation of sophisticated VRP models, providing insights for optimizing urban logistics in increasingly dynamic and customer-driven environments.

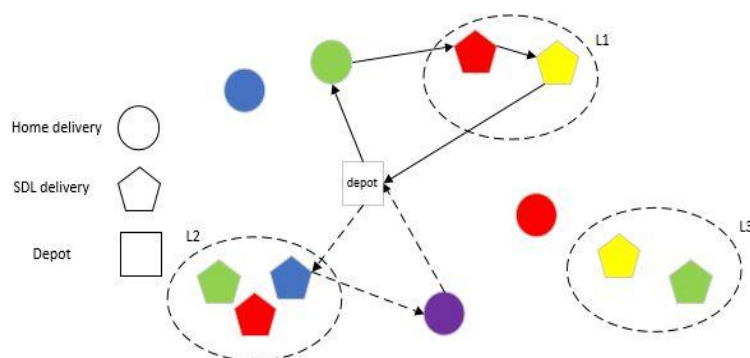


Fig. 1. Vehicle routing problem with delivery options considering shared delivery centers.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on VRPs. Section 3 presents the mathematical formulation and methodology. Section 4 discusses the numerical example and computational results, and finally, Section 5 concludes the study with managerial implications and future research directions.

2 | Literature Review

2.1 | Classical Vehicle Routing Problem

Dantzig and Ramser [5] first proposed the VRP as an expansion of the traditional Traveling Salesman Problem (TSP). VRP seeks to optimize the routes of multiple vehicles with the goal of minimizing distribution costs while meeting customer demands, in contrast to TSP, which aims to minimize the total distance traveled by a single salesman visiting multiple cities. Initially applied to optimize fuel delivery routes, VRP has since become a core issue in logistics and operations research, serving as the foundation for many practical applications, such as waste collection, urban logistics, and e-commerce delivery [6].

2.2 | Extensions of the Vehicle Routing Problem

The traditional VRP has evolved along several dimensions over time to address real-world challenges. These include the Capacitated Vehicle Routing Problem (CVRP), which ensures each vehicle does not exceed its load limit [2]; Heterogeneous Fleet Vehicle Routing Problem (HFVRP), which considers vehicles with varying capacities and costs [6]; Multi-Depot Vehicle Routing Problem (MDVRP), which models multiple warehouses for dispatch [7]; and Vehicle Routing Problem with Time Windows (VRPTW), which guarantees customer service within predetermined periods [8].

Additionally, in response to customer convenience demands and delivery challenges in urban areas, new dimensions have emerged. One such dimension is Vehicle Routing Problem with Roaming Delivery Locations (VRPRDL), which allows customers to designate multiple potential delivery locations based on their daily movements. Another dimension is the Vehicle Routing Problem with Delivery Options (VRPDO), which allows deliveries to Shared Delivery Centers (SDCs) or to personal addresses, increasing flexibility and reducing distribution costs [9]. Examples of typical variations are illustrated in Fig. 2, showing how VRP adapts to industry demands.

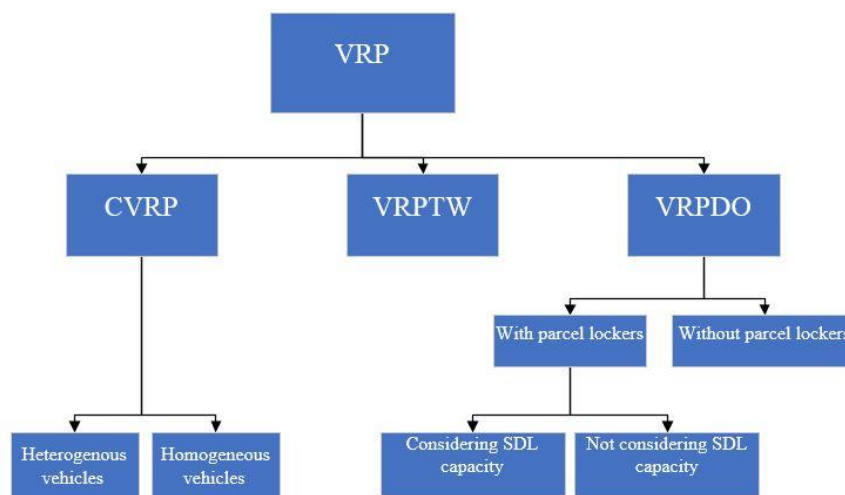


Fig. 2. Some important variants of the vehicle routing problem.

2.2.1 | Vehicle routing problem with roaming delivery locations

The idea of VRP with roaming delivery locations was introduced to address unsuccessful delivery attempts resulting from customers' absence at designated times and locations. One method allows customers to specify a variety of locations with corresponding time windows for order delivery throughout the day [4]. This flexibility enables logistics planners to select the most cost-effective location–time combination, improving delivery reliability and reducing expenses. The model is further enhanced by incorporating stochastic travel

times, which are handled using a combination of a Monte Carlo method and a Greedy Randomized Adaptive Search Procedure (GRASP) [10]. Another roaming location option that has been explored is in-car delivery, which has the potential to reduce costs by up to 20% [11]. Subsequent research considered electric vehicle fleets to address environmental concerns [12] and incorporated uncertainties in customer plans and travel times [13], [14].

2.2.1 | Vehicle routing problem with shared delivery locations

Another notable dimension is VRP with SDLs, or SDL-integrated VRP. This approach addresses failed deliveries and high operational costs by allowing customers to pick up their packages from parcel lockers or pickup locations [4]. It reduces route lengths and enhances environmental sustainability by combining deliveries to residential addresses with consolidated drop-offs at parcel lockers [9]. However, requiring self-pickup may lead customers to perceive a lower service quality, necessitating financial or in-kind compensation from businesses [9], [15].

Recent research integrates heterogeneous locker capacities [13] and models combined home and locker deliveries under capacity constraints [16], demonstrating that such hybrid delivery models can improve cost efficiency by up to 40% [14]. This combined delivery policy has proven effective in balancing customer satisfaction and operational costs in urban last-mile distribution.

3 | Methodology

3.1 | Problem Description

This research addresses a single depot VRP with private Roaming and SDLs (VRP-RSD). The problem is represented as a graph $G = (N, A)$, where N is the set of nodes, and A is the set of arcs connecting them. The node set N consists of the depot node and the customer delivery option nodes. The depot node is denoted D and represents the central dispatching point from which vehicles depart and to which they return after completing their routes.

Customer delivery options are represented by the set L , which includes all possible delivery locations that customers can select, such as their home address or a nearby shared delivery location. For each customer c , the subset $L_c \subseteq L$ defines the available delivery options specifically assigned to them. Furthermore, the set $S \subseteq L$ includes shared delivery locations with limited capacity, such as parcel lockers or micro-hubs.

The arc set A is defined as all possible connections between pairs of distinct nodes (i, j) , where each arc represents a potential travel route from node i to node j , with an associated travel time t_{ij} calculated based on the Euclidean distance and an assumed fixed average vehicle speed.

A fleet of heterogeneous vehicles, represented by set K , is available to serve customer orders. Each vehicle departs from the depot, visits assigned delivery locations to serve customer demands, and returns to the depot within a maximum route duration. Vehicles have identical capacity limits, and customer demands are deterministic and must be delivered entirely to their selected delivery option. Deliveries must occur within the specified time windows.

$[a_l, b_l]$ of each delivery option $l \in L$, with waiting incurred if vehicles arrive before the time window start.

The objective is to determine optimal delivery option selections for customers, feasible vehicle routes covering all selected options, and scheduling decisions that minimize total operational cost. This cost consists of travel cost, waiting cost due to early arrivals, and penalties incurred when customers are assigned to shared delivery locations instead of direct home delivery. Problem assumption

3.2 | Problem Assumption

The following assumptions are made to formulate the problem:

- I. Vehicles are heterogeneous in capacity but share identical operational characteristics such as speed and service time.
- II. Travel times between nodes are calculated based on Euclidean distances and a fixed average vehicle speed.
- III. Service time is fixed and identical across all delivery points.
- IV. Each customer must be assigned to exactly one delivery option, either home or a shared location.
- V. Using SDLs incurs an operational or customer dissatisfaction penalty.
- VI. Each SDL has a limited capacity, constraining the total demand or number of customers it can serve in a delivery cycle.
- VII. Vehicles depart from and return to the depot at most once within the planning horizon.
- VIII. Time windows are defined for each delivery option, and deliveries must occur within these windows; waiting time is incurred if vehicles arrive earlier.

3.3 | Problem Formulation

In this section, the sets and parameters used in the mathematical model are specified in *Tables 1* and *2*, respectively. The decision variables used in the mathematical model are presented in *Table 3*.

Table 1. The sets used in the mathematical model.

Symbol	Definition
C	Set of customers
L	Set of all delivery options across customers
L_c	Set of delivery options available for customer c
S	Set of SDL
K	Set of vehicles
N	Set of all nodes
D	Central depot node from which all vehicles depart and to which they return

Table 2. The parameters used in the mathematical model.

Parameter	Description
t_{ij}	Travel time between nodes i and j
d_{ij}	Distance between nodes i and j
Q^k	Capacity of vehicle k
cap_s	Capacity of shared locations $s \in S$
q_l	Demand associated with delivery option l
$[a_i, b_i]$	Earliest and latest possible time to visit node i
c_d	Cost per kilometer traveled.
c_w	Cost per hour of waiting time
c_s	Penalty for using SDLs
st	Fixed service time at each node
M	A large constant (big-M)
Du	Maximum allowed tour duration

Table 3. Decision variables.

Decision variable	Definition
x_{ij}^k	A binary variable, being one, means that the vehicle k visited j after i
y_{cl}	A binary variable, being one, means that customer c is assigned to delivery option l
z_{cs}	A binary variable, being one, means customer c is assigned to shared location s
δ_l	Arrival time at location l
w_l	Waiting time at location l

$$\min \sum_{k \in K} \sum_{(i,j) \in N} c_d d_{ij} x_{ij}^k + \sum_{l \in L} c_w w_l + \sum_{c \in C} \sum_{s \in S \cap L_c} c_s z_{cs}. \quad (1)$$

$$\sum_{l \in L_c} y_{cl} = 1, \quad \forall c \in C. \quad (2)$$

$$z_{cs} = y_{cs}, \quad \forall c \in C, \forall s \in S \cap L_c. \quad (3)$$

$$\sum_{c \in C} q_s z_{cs} \leq \text{cap}_s, \quad \forall s \in S. \quad (4)$$

$$a_l \leq \delta_l + w_l, \quad \forall l \in L. \quad (5)$$

$$\delta_l \leq b_l, \quad \forall l \in L. \quad (6)$$

$$\delta_j \geq \delta_i + t_{ij} + w_i + st - M(1 - x_{ij}^k), \quad \forall i, j \in L, \quad \forall k \in K. \quad (7)$$

$$\sum_{i \in N, i \neq l} \sum_{k \in K} x_{il}^k = y_{cl}, \quad \forall c \in C, \quad \forall l \in L_c. \quad (8)$$

$$\sum_{j \in N} x_{ij}^k = \sum_{j \in N} x_{ji}^k, \quad \forall i \in L, \quad \forall k \in K. \quad (9)$$

$$\sum_{j \in L} x_{Dj}^k \leq 1, \quad \forall k \in K. \quad (10)$$

$$\sum_{j \in L} x_{jD}^k \leq 1, \quad \forall k \in K. \quad (11)$$

$$\sum_{(i,j) \in N} t_{ij} x_{ij}^k \leq Du, \quad \forall k \in K. \quad (12)$$

$$\sum_{c \in C} \sum_{l \in L_c} q_l y_{cl} \leq Q^k, \quad \forall k \in K. \quad (13)$$

$$x_{ij}^k \in \{0,1\}, \quad \forall i, j \in N, \quad \forall k \in K. \quad (14)$$

$$y_{cl} \in \{0,1\}, \quad \forall c \in C, \quad \forall l \in L_c. \quad (15)$$

$$z_{cs} \in \{0,1\}, \quad \forall c \in C, \quad \forall s \in S \cap L_c. \quad (16)$$

$$\delta_l \geq 0, \quad \forall l \in L. \quad (17)$$

$$w_l \geq 0, \quad \forall l \in L. \quad (18)$$

The objective *Function (1)* minimizes the total operational cost, which includes travel costs across all vehicle routes, waiting time costs at delivery locations, and penalties incurred for using shared delivery points. *Eq. (2)* ensures that each customer selects exactly one delivery option among their feasible alternatives. *Eq. (3)* enforces that if a customer is assigned to an SDL, the assignment variable is activated accordingly. *Eq. (4)* imposes the capacity restriction for SDLs to ensure that their total assigned demand does not exceed the

allowed capacity. Eqs. (5) and (6) ensure feasibility with respect to delivery time windows by enforcing that service starts within the specified time interval. Eq. (7) is the time propagation constraint, which guarantees that if a vehicle travels from one node to another, the arrival time at the next node accounts for travel time, waiting time, and service time. Eq. (8) ensures that if a delivery option is selected for a customer, exactly one incoming arc is assigned to it. Eq. (9) maintains flow conservation, ensuring that the number of incoming and outgoing arcs at each visited node is balanced. Eqs. (10) and (11) limit each vehicle to depart from and return to the depot at most once. The restriction limiting each vehicle's cumulative travel time to its predefined maximum allowable duration is mathematically formulated in Eq. (12). Eq. (13) enforces the vehicle capacity constraint, ensuring that the total served demand does not exceed vehicle capacity. Finally, Eqs. (14)–(18) define the types and ranges of the decision variables used in the model.

4 | Numerical Example

To evaluate the operational impact of delivery option flexibility in single-depot VRPs, two scenarios were formulated and solved using a mixed-integer programming model implemented in Python with PuLP. Both scenarios utilized a heterogeneous fleet of five vehicles with capacities fixed at 100, 120, 90, 110, and 95 units for vehicles K0 through K4, respectively. The depot was located at (50, 50), and the vehicle speed was assumed to be 60 km/h. A service time of 10 minutes was imposed at each customer location to account for loading and unloading. Additionally, cost parameters were set at 30 currency units per kilometer traveled, 100 units per hour of waiting time, and a penalty of 50 units for using shared delivery options to capture potential coordination costs. The maximum allowed route duration was set at 1000 minutes.

In the first scenario, each customer was allowed to have multiple delivery options, enabling the model to select the optimal delivery node to minimize operational costs. The results showed an objective value of 5444.69 units, with only 3 vehicles serving all 6 customers. Additionally, one SDL was used, and no waiting costs were incurred. This outcome is illustrated in Fig. 3, which depicts the optimized routes and selected delivery points under flexible options. The results indicate that offering multiple delivery options enables effective route consolidation, thereby reducing both total travel costs and the number of vehicles needed.

In contrast, the second scenario considered a system where each customer was assigned only a single delivery location. Although the model allowed SDCs, in this dataset, only the first customer's delivery location was designated as a shared center, while the others had no alternative. The results showed a substantially higher objective value of 9183.21 units, with all five vehicles required to serve the same six customers. No shared deliveries were used beyond the predefined shared node, and no waiting costs were incurred. This outcome is shown in Fig. 4, where restricted delivery options led to longer total routes and higher fleet utilization. The significant increase in operational cost demonstrates the inefficiency that arises when customers are restricted to a single delivery option, even if shared centers are permitted in the system.

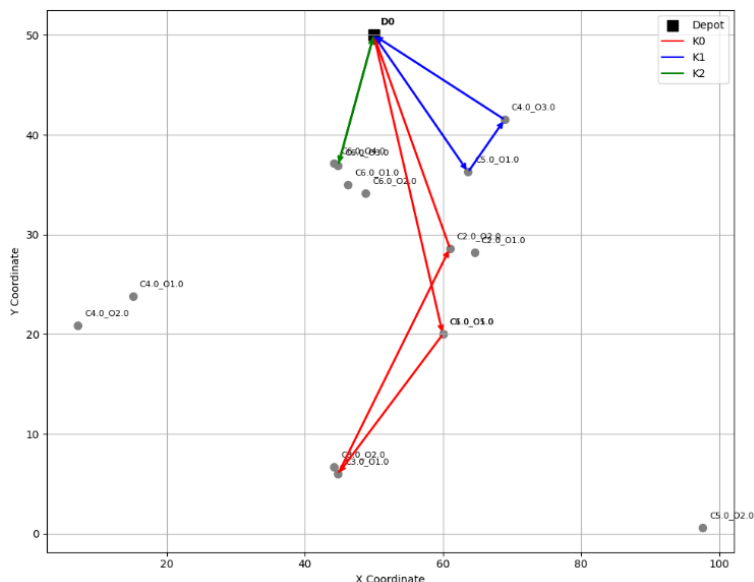


Fig. 1. Delivery routes with multiple options per customer.

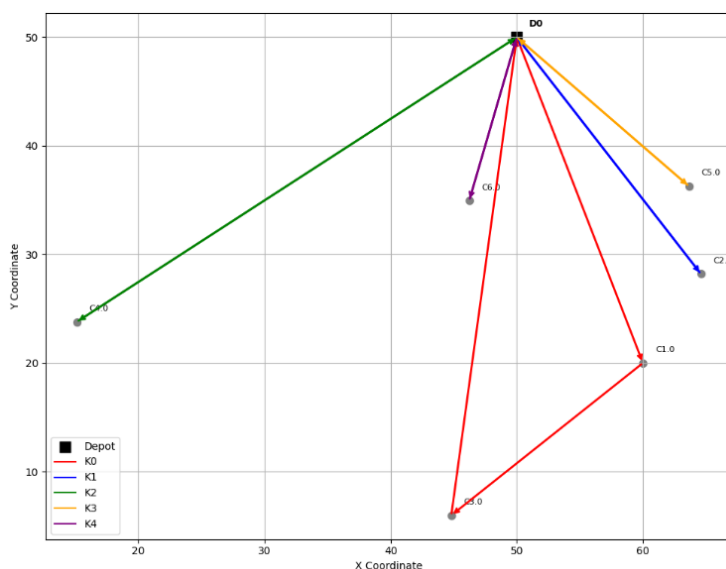


Fig. 2. Delivery routes with a single option per customer.

Overall, these findings demonstrate the critical impact of delivery flexibility on vehicle routing performance. For instance, allowing multiple delivery options per customer reduced total operational costs by over 40% and led to a marked decrease in fleet utilization compared to the single-option scenario. These results highlight the managerial importance of integrating customer flexibility into last-mile delivery planning to achieve substantial efficiency gains in urban logistics networks.

5 | Conclusion

The VRP with private Roaming and SDLs (VRP-RSD) was examined in this study, accounting for SDCs, customer delivery choices, and heterogeneous fleet capacities. The model incorporated realistic features such as alternative customer delivery locations, time-window restrictions, service times, and vehicle and shared-facility capacity limitations by developing a mixed-integer linear programming formulation. By optimizing

vehicle routing, cutting operational costs, and minimizing fleet requirements, the results showed that adding flexibility through shared delivery points and multiple delivery options can improve overall system efficiency.

To achieve sustainable, cost-effective last-mile operations, the proposed model emphasizes the importance of combining customer choice with cooperative delivery infrastructure. It offers useful insights for urban logistics planning. However, advanced heuristic and metaheuristic solution approaches are advised for practical implementation due to the computational complexity of solving such detailed models, particularly in large-scale urban contexts. Methods such as large neighborhood search, in conjunction with hybrid evolutionary algorithms, can effectively address scalability issues without sacrificing solution quality.

Overall, by demonstrating how delivery flexibility and shared resource utilization can significantly impact performance, this study advances the field of last-mile logistics optimization. It illustrates how operational guidelines that allow multiple delivery options for each customer can significantly reduce fleet size and routing costs while simultaneously enhancing customer satisfaction and delivery reliability. To increase the work's practical applicability in rapidly changing urban environments, future research should account for stochastic travel times, dynamic demand, and integration with real-time vehicle routing systems. Furthermore, integrating environmental goals, such as reducing carbon emissions, and considering the use of electric vehicles may offer a more comprehensive strategy for sustainable urban distribution planning.

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Data Availability

The data used in this study were generated and processed by the authors for numerical analyses and model validation. The datasets supporting the findings of this study are available from the corresponding author upon reasonable request.

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