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Developing A Sustainable Supply Chain Model Considering Uncertain Conditions in the Dairy Industry

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Abstract

A large volume of food and perishable products is transferred daily from suppliers to manufacturers and eventually to retailers in the supply chain. Due to the perishable nature of these products and the associated storage limitations, production and transportation costs fluctuate, causing significant environmental impacts. Sustainability and uncertainty issues in perishable products require further research. This study presents a mathematical model for managing a multi-period, multi-product sustainable supply chain in the dairy industry, considering uncertainty. A mixed-integer nonlinear programming model is developed to optimize total supply chain costs, with a focus on the reuse of recovered products. The model is solved using a scenario-based stochastic programming approach and the Mulvey method, which accounts for uncertainties in demand parameters. The results indicate the proposed model performs effectively, providing high-quality solutions within a reasonable timeframe. These findings offer a valuable tool for decision-making aimed at reducing costs and minimizing negative environmental and social impacts through product recovery and reuse.

Keywords: Perishable products, Uncertainty, Sustainable supply chain, Robust optimization, Stochastic optimization, Value of stochastic solution, Expected value of perfect information indices.

1 | Introduction

The proposed applied model consists of a five-echelon sustainable supply chain, comprising suppliers, hybrid production centers, hybrid warehouse-picking centers, retailers, and product disposal and recovery centers designed for short-life products such as dairy industries [1]. This model is formulated as a multi-period and multi-product model, and a percentage of the recovered product is incorporated as raw material supplied by suppliers. The retailers' demand is considered uncertain, and the relationship between production and inspection centers, as well as between warehouse and picking centers, is modeled in an integrated manner. To achieve robust optimization, various approaches such as scenario generation and the Malevolent (Malve) approach have been employed. The proposed model is presented as a linear mixed-integer programming

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model. Given the storage limitations and the limited shelf life of dairy products, optimal scheduling and transportation are of great importance [2]. Products at the warehouse-picking centers, due to their limited shelf life, are returned to the production centers if not transferred within a specified time window. The demand of each retailer for each product type in each period is predetermined, and the model is designed to maximize responsiveness to retailer demand. The model has been solved using the CPLEX solver of GAMS software version 23.5 for problems of various sizes, and the results are presented for different test problems. Subsequently, by varying the values of different model parameters, the performance of the model has been evaluated, and a sensitivity analysis has been conducted [3], [4].

2 | Figures and Tables

The case study examined in this thesis is inspired by a food industry in which various short-life products are manufactured and stored for a limited period. After production and temporary storage, the products are shipped to warehouse centers (based on a FIFO policy) and ultimately delivered to customers. In addition, a percentage of returned products, after undergoing an inspection process, are either discarded or sent to other industries for recovery. In other words, the remaining percentage of recovered returned goods is reused in the forward flow. Accordingly, the problem considered in this research is a multi-echelon closed-loop supply chain with forward and reverse flows designed for a network dealing with short-life products. The considered echelons include suppliers, hybrid production and inspection centers, hybrid warehouse and picking centers, as well as disposal and recovery centers, all established based on the requirements of short-life products. The main decision in this model is defined with the objective of reducing costs and minimizing waste. The most significant contribution of this research is the consideration of the MFCA technique with the aim of waste reduction. In this technique, waste is costed on a weight basis. For certain food products, it is possible to convert a percentage of the products into another product; however, this is not feasible for the entire percentage of expired products [5], [6].

Ordering costs and holding costs in the proposed model are subject to the possibility of placing orders or reserving raw materials, after which the materials are sent to the hybrid warehouse and picking center for later use of the goods. Given the characteristics of short-life products, the products may remain at the HWC centers or, in case the products are not dispatched from the Hybrid Warehouse Collection (HWC) centers to the retailers within the maximum allowed period, be returned to the Hybrid Production Inspection (HPI) centers after a specific period of time. Due to the short-life nature of the products, each time period is considered to be one day.

In the proposed model, the objective is to maximize the fulfillment of retailers' demand such that the total network profit is maximized, while the recoverable waste and the waste destined for disposal are minimized. Due to the nature of short-life products, a situation may arise in which the time required for a retailer to receive the products exceeds the maximum coverage time. Under such conditions, the retailer is not considered in the model, as its demand cannot be met [7].

Furthermore, a penalty is imposed for unmet demands. In the reverse flow of the studied problem, defective products are returned both from the retailers and from the HWC centers. Those returned from the retailers are first sent to the HWC centers and, in the same period, immediately transferred to the HPI centers. After the inspection process, certain portions of the returned products are recovered, and the remaining portions are disposed of. Therefore, in this research, since a percentage of the recoverable products is sent to the recovery centers, the revenue from recovery is considered in place of the recovery and HPI costs, and the transportation costs are replaced by costs to the recovery centers. Moreover, a percentage of the recovered products is returned to the suppliers as input. In what follows, *Figs. 1* and *2* illustrate the difference introduced [8].

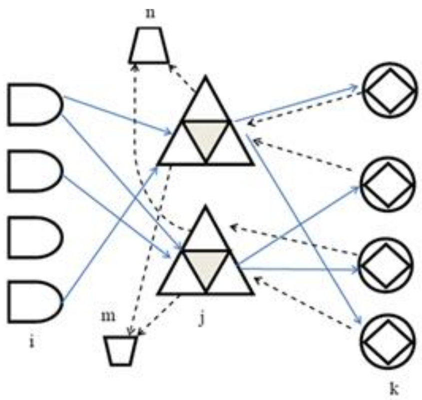

Fig. 1. Initial model proposed by researchers.

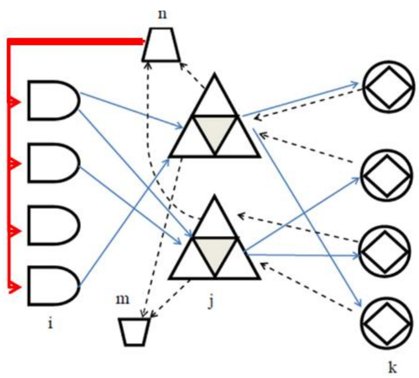

Fig. 2. The proposed research model.

The assumptions of the problem are as follows:

I. The number and locations of HPI centers, as well as the maximum number and locations of potential suppliers, HWC centers, retailers, recovery centers, and disposal centers, are fixed and known in advance.

II. The proposed model is developed as a multi-period and multi-product model [9].

III. Retailers' demand is assumed to be uncertain [10].

IV. All facilities, except for the raw material warehouse at the HPI center, are subject to limited capacity constraints.

V. It is assumed that both the HWC center and the raw material warehouse at the HPI center incur inventory holding costs [11].

VI. Due to the short life cycle of the products, they remain at HWC centers for a maximum of three periods. If no demand arises during this time, the products are returned after the specified periods.

VII. Each HWC center is responsible for the collection and storage of returned products [12].

Tables 1-3 present the indices, parameters, and decision variables of the mathematical model [1].

2.1 | Variables and Equations

Table 1. Sets used in the stochastic mathematical model.

T	Set of planning periods
I	Set of suppliers
J	Set of manufacturers
K	Set of warehouses
L	Set of retailers
M	Set of disposal centers
N	Set of recycling/recovery centers
c	Set of products
R	Set of raw materials or components required for product manufacturing
s	Set of scenarios

Table 2. Parameters used in the stochastic mathematical model.

MS_t	Maximum number of suppliers that can be selected in period t
MW_t	Maximum number of warehouses that can be established/selected in period t
MD_t	Maximum number of disposal centers that can be selected in period t
MR_t	Maximum number of recycling centers that can be selected in period t
FS_{it}	Fixed cost of selecting supplier i in period t
Fw_{kt}	Fixed cost of establishing warehouse k in period t
FD_{mt}	Fixed cost of selecting disposal center m in period t
FR_{nt}	Fixed cost of selecting recycling center n in period t
FO_{rit}	Fixed ordering cost of raw material/component r from supplier i in period t
BC_{irt}	Purchasing cost of one unit of raw material/component r from supplier i in period t
PC_{jct}	Production and packaging cost of one unit of product c by manufacturer j in period t
DC_{mct}	Disposal cost of one unit of product c at disposal center m in period t
HC_{kct}	Inventory holding cost of one unit of product c at warehouse k in period t
Φ_{lct}	Penalty cost of unmet demand for product c at retailer l in period t
IC_{jct}	Inspection cost per unit of product c at manufacturing center j in period t
HCP_{rjt}	Inventory holding cost per unit of raw material r at manufacturing center j in period t
$TCSP_{ijrt}$	Transportation cost of one unit of raw material/component r from supplier i to manufacturer j in period t
$TCSP1_{nitc}$	Transportation cost of one unit of recycled material c from recycling center n to supplier i in period t
$TCPW_{jkct}$	Transportation cost of product c from manufacturer j to warehouse k in period t
$TCPD_{jmct}$	Transportation cost of product c from manufacturer j to disposal center m in period t
$UCAPP_{jc}$	Capacity utilization required for producing one unit of product c by manufacturer j
$UCAPD_{mc}$	Capacity utilization required for disposing one unit of product c at disposal center m
$UCAPR_{nc}$	Capacity utilization required for recycling one unit of product c at recycling center n
$CAPS_{ir}$	Supply capacity of raw material/component r provided by supplier i
$CAPP_{jc}$	Production capacity of manufacturer j for product c
$CAPW_k$	Capacity of warehouse k for storing all products
$CAPD_m$	Capacity of disposal center m
$CAPR_n$	Capacity of recycling center n
r_{1c}	Return rate of product c from retailers to warehouses
r_{2c}	Recovery rate of returned defective product c
r_{3c}	Return rate of recovered product c as input to suppliers
D_{lct}	Demand of retailer l for product c in period t
$UCAPS_{rc}$	Conversion coefficient of product c into raw material r
rv_{ct}	Revenue generated from the sale of product c in period t
RVR_{nct}	Revenue generated from recycling one unit of product c at recycling center n in period t
Vc	Volume of one unit of product c
TWRkl	Transportation time required to deliver products from the integrated warehouse–collection center k to retailer l
EXFt	Maximum allowable delivery time from integrated warehouse–collection centers to retailers in period t
PS	Probability of occurrence of each scenario

Table 3. Decision variables used in the stochastic mathematical model.

SS_i	Binary variable equal to 1 if supplier i is selected; 0 otherwise
SW_k	Binary variable equal to 1 if warehouse k is selected; 0 otherwise.
SD_m	Binary variable equal to 1 if disposal center m is selected; 0 otherwise.
SR_n	Binary variable equal to 1 if recycling center n is selected; 0 otherwise.
SC_{klt}	Binary variable equal to 1 if retailer l is covered by warehouse k in period t; 0 otherwise.
SO_{ijrt}	Binary variable equal to 1 if raw material/component r is ordered from supplier i by manufacturer j in period t; 0 otherwise.
X_{ijrt}	Quantity of raw material/component r transported from supplier i to manufacturer j in period t.
$X1_{nitc}$	Quantity of recycled material c transported from recycling center n to supplier i in period t, where this quantity equals r3c multiplied by the total amount of products entering the recycling center.
Y_{jkct}	Quantity of product c transported from manufacturer j to warehouse k in period t.
RYT_{kjct}	Quantity of product c returned from warehouse k to manufacturer j in period t due to the expiration of its useful life.
Z_{klct}	Quantity of product c transported from warehouse k to retailer l in period t.
RZ_{lkct}	Quantity of returned product c transported from retailer l to warehouse k in period t.
RY_{kjct}	Quantity of returned product c received at collection center k from retailers and subsequently sent to manufacturer j within the same period t.
RR_{jnct}	Quantity of product c transported from manufacturer j to recycling center n in period t.
RD_{jmct}	Quantity of product c transported from manufacturer j to disposal center m in period t.
SP_{rjt}	Inventory level of raw material r at manufacturer j in period t.
$S_{kct't}$	Inventory level of product c produced in period t' and remaining in warehouse k at the end of period t.
UD_{lct}	Unmet demand of retailer l for product c in period t.
mn_j	Auxiliary variable used for linearization purposes.

2.2 | Objective Functions and Constraints

$$\begin{aligned} \mathrm{Max}z = & \sum_t\sum_k\sum_l\sum_c rv_{ct}\,.\,Z_{klct} + \sum_t\sum_k\sum_l\sum_c RVR_{nct}\,.\,RR_{jnct} - \sum_t\sum_c\sum_l \varphi_{lct}\,.\,UD_{lct} \\ & - \sum_t\sum_r\sum_i\sum_j FO_{rit}\,.\,SO_{ijrt} - \sum_t\sum_r\sum_i\sum_j BC_{irt}\,.\,X_{ijrt} - \sum_t\sum_j\sum_r HCP_{rjt}\,.\,SP_{rjt} \\ & - \sum_t\sum_c\sum_k\sum_j PC_{jct}\,.\,Y_{jkct} - \sum_t\sum_c\sum_k\sum_j HC_{kct}\,.\,S_{kct't} \\ & - \sum_i\sum_j\sum_r\sum_t TCSP_{ijrt}\,.\,X_{ijrt} - \sum_i\sum_j\sum_r\sum_t TCSP1_{nitc}\,.\,X1_{nitc} \\ & - \sum_j\sum_k\sum_c\sum_t TCPW_{jkct}\,.\,(RYT_{kjct} + RY_{kjct} + Y_{jkct}) \\ & - \sum_k\sum_l\sum_c\sum_t TCWR_{klct}\,.\,(Z_{klct} + RZ_{lkct}) - \sum_j\sum_m\sum_c\sum_t TCPD_{jmct}\,.\,RD_{jmct} \\ & - \sum_{t>3}\sum_k\sum_l\sum_c rv_{ct-3}\,.\,RZ_{lkct} - \sum_j\sum_k\sum_c\sum_t IC_{jct}\,.\,(RYT_{kjct} + RY_{kjct}) \\ & - \sum_j\sum_m\sum_c\sum_t DC_{mct}\,.\,RD_{jmct}. \end{aligned} \tag{1}$$

Eq. (1) expresses the difference between the total revenues and total costs of the supply chain, there by maximizing the overall profit.

$$\sum_j X_{ijrt} + X1_{nict} \le CAPS_{ir}\,.\,SS_i, \qquad \forall i, n, c, \forall r, \forall t. \tag{2}$$

Constraint (2) represents the capacity limitation of each supplier for providing raw materials in each period.

$$\sum_k Y_{jkct}\,.\,UCAPP_{jc} \le CAP.P_{jc}, \qquad \forall c, \forall t, \forall j. \tag{3}$$

Constraint (3) ensures that the quantity of each product produced by a manufacturer in a given period does not exceed the manufacturer's available production capacity.

$$\sum_{c}\sum_{j} \mathrm{UCAPD}_{mc}.\mathrm{RD}_{jmct} \le \mathrm{CAPD}_{m}.\mathrm{SD}_{m}, \qquad \forall m, \forall t. \tag{4}$$

Constraint (4) ensures that the quantity of each product sent to a disposal center in a given period does not exceed the center's disposal capacity.

$$\sum_{e}\sum_{j} \mathrm{UCAPR}_{nc}.\mathrm{RR}_{jnct} \le \mathrm{CAPR}_{n}.\mathrm{SR}_{n}, \qquad \forall n, \forall t. \tag{5}$$

Constraint (5) ensures that the quantity of products processed at each recycling center does not exceed its available recycling capacity.

$$\sum_{c}\sum_{j} \mathrm{V}_{c}.\mathrm{Y}_{jkt} \le \mathrm{CAPW}_{k}.\mathrm{SW}_{k}, \qquad \forall k, t = 1. \tag{6}$$

Constraint (6) ensures that the quantity of products stored in each warehouse does not exceed its available storage capacity.

$$\sum_{c} \mathrm{V}_{c}.\left(\mathrm{S}_{kx,t-1,t-1} + \sum_{j} \mathrm{Y}_{jktt}\right) \le \mathrm{CAPW}_{k} \cdot \mathrm{SW}_{k}, \qquad \forall k, t = 2. \tag{7}$$

$$\sum_{c} \mathrm{V}_{c} \cdot \left(\mathrm{S}_{k,t,-2,t-1} + \mathrm{S}_{kk,t-1,t,-1} + \sum_{j} \mathrm{Y}_{jkct}\right) \le \mathrm{CAPW}_{k} \cdot \mathrm{SW}_{k}, \qquad \forall k, t = 3. \tag{8}$$

$$\sum_{c} \mathrm{V}_{c} \cdot \left(\mathrm{S}_{kx,t-3,t-1} + \mathrm{S}_{kx,t-2,t-1} + \mathrm{S}_{kk,t-1,t-1} + \sum_{j} \mathrm{Y}_{jkkt}\right) \le \mathrm{CAPW}_{k}\,\mathrm{SW}_{k}, \qquad \forall k, t \ge 4. \tag{9}$$

Constraints (7)-(9) impose the warehouse capacity restrictions by ensuring that the aggregate storage space occupied by incoming products and inventories remaining from previous periods remains within the warehouse capacity. These constraints also enforce a maximum product holding time of three periods.

$$\sum_{j} \mathrm{RYT}_{kjct} = 0, \qquad \forall k, \forall c, t = 1. \tag{10}$$

$$\mathrm{S}_{kctt} = \max\left(\sum_{j} \mathrm{Y}_{jkct} - \sum_{T} \mathrm{Z}_{klct}, 0\right), \qquad \forall k, \forall c, t = 1. \tag{11}$$

$$\sum_{j} \mathrm{RYT}_{kjct} = 0, \qquad \forall k, \forall c, t = 2. \tag{12}$$

$$\mathrm{S}_{k,t,-1,t} = \max\left(\mathrm{S}_{kc,t-1,t-1} - \sum_{i} \mathrm{Z}_{klct}, 0\right), \qquad \forall k, \forall c, t = 2. \tag{13}$$

$$\mathrm{S}_{kctt} = \max\left(\sum_{j} \mathrm{Y}_{jkct} + \min\left(\mathrm{S}_{kc,t-1,t-1} - \sum_{i} \mathrm{Z}_{klct}, 0\right), 0\right), \qquad \forall k, \forall c, t = 2. \tag{14}$$

$$\sum_{j} \mathrm{RYT}_{kjct} = 0, \qquad \forall k, \forall c, t = 3. \tag{15}$$

$$\mathrm{S}_{kc,t-2,t} = \max\left(\mathrm{S}_{kc,t-2,t-1} - \sum_{t} \mathrm{Z}_{klct}, 0\right), \qquad \forall k, \forall c, t = 3. \tag{16}$$

$$S_{kctt} = \max\left(\sum_j Y_{jkct} + \min\left(S_{kc,t-1,t-1} + \min\left(S_{kc,t-2,t-1} - \sum_t Z_{klct}, 0\right), 0\right), 0\right), \quad \forall k, \forall c, t = 3. \tag{17}$$

$$\sum_j RYT_{kjct} = \max\left(S_{kc,t-3,t-1} - \sum_T Z_{klct,}, 0\right), \qquad \forall k, \forall c, t \geq 4. \tag{18}$$

$$S_{kc,t-2,t} = \max\left(S_{kc,t-2,t-1} + \min\left(S_{kc,t-3,t-1} - \sum_t Z_{klct}, 0\right), 0\right), \quad \forall k, \forall c, t \geq 4. \tag{19}$$

$$\begin{aligned} S_{kc,t-1,t} = \max\Bigg(& S_{kct-1,t-1} \\ & + \min\Bigg(S_{kc,t-2,t-1} \\ & + \min\left(S_{kc,t-2,t-1} + S_{kc,t-3,t-1} - \sum_l Z_{kict}, 0\right), 0\Bigg), 0\Bigg), \quad \forall k, \forall c, t \geq 4. \end{aligned} \tag{20}$$

$$\begin{aligned} S_{kct} = \max\Bigg(& \sum_j Y_{jkt} \\ & + \min\Big(S_{kc,t-1,t-1} \\ & + \min\Big(S_{kc,t-2,t-1} \\ & + \min\left(S_{kc,t-3,s-1} - \sum_1 Z_{kct}, 0\right), 0\Big), 0\Big), 0\Bigg), \quad \forall k, \forall c, t \geq 4. \end{aligned} \tag{21}$$

Constraints (10)-(21) track the inventory of products stored in the warehouse and identify the quantities that must be returned upon reaching the end of their allowable storage life. Given that the maximum product lifetime is limited to three periods, products that entered the warehouse three periods earlier and remain unsold due to the absence of demand are returned to the manufacturer.

$$\sum_i X_{ijrt} - \sum_k \sum_c Y_{jkct}.UCAPS_{rc} = SP_{rjt}, \qquad \forall j, \forall r, t = 1. \tag{22}$$

$$\sum_i \left(X_{ijrt} + \sum_{C,K,L} r3c\, RZ_{lkct}\right) - \sum_k \sum_c Y_{jkct}.UCAPS_{rc} + SP_{rj,t-1} = SP_{rjt}, \quad \forall j, \forall r, \forall t \geq 2. \tag{23}$$

Constraints (22) and *(23)* establish the balance relationship between suppliers and manufacturers while accounting for the inventory of raw materials and components at the manufacturing facilities.

$$M \cdot SO_{ijrt} \geq X_{ijtr}, \qquad \forall i, \forall r, \forall r, \forall t. \tag{24}$$

Constraint (24) links the continuous flow variable X to the binary ordering decision variable SO. Whenever a positive quantity is ordered, SO is activated and takes the value 1; otherwise, SO remains 0. This formulation ensures that the fixed ordering cost is incurred only when an order is placed, as reflected in the objective function.

$$\sum_l Z_{klct} \leq \sum_j Y_{jkct}, \qquad \forall c, \forall k, t = 1. \tag{25}$$

$$\sum_l Z_{klct} \leq \sum_j Y_{jkct} + S_{kc,t-1,t-1}, \qquad \forall c, \forall k, t = 2. \tag{26}$$

$$\sum_l Z_{klct} \leq \sum_j Y_{jkct} + S_{kc,t-1,t-1} + S_{kc,t-2,t-1}, \qquad \forall c, \forall k, t = 3. \tag{27}$$

$$\sum_{l} Z_{klct} \leq \sum_{j} Y_{jkct} + S_{kc,t-1,t-1} + S_{kk,t-2,t-1} + S_{kc,t-3,t-1}, \qquad \forall c, \forall k, t \geq 4. \tag{28}$$

Constraints (26)-(28) ensure that the total quantity of products shipped from each warehouse to all retailers does not exceed the total amount of products received at that warehouse from all manufacturers. This balance accounts for both the products arriving in the current period and the inventory carried over from previous periods.

$$\sum_{l} Z_{klct} \leq \sum_{j} Y_{jkct} + S_{kc,t-1,t-1}, \qquad \forall c, \forall k, t = 2. \tag{29}$$

$$SC_{klt} . TWR_{kl} \leq EXF_t, \qquad \forall k, \forall l, \forall t. \tag{30}$$

$$EXF_t \cdot (1 - SC_{klt}) \leq TWR_{kl}, \qquad \forall k, \forall l, \forall t. \tag{31}$$

Constraints (29)-(31) enforce the retailer coverage requirements. Retailers whose service time exceeds the maximum allowable threshold are regarded as uncovered, and their demand is therefore not considered in the model. In contrast, retailers located within the specified service coverage radius are considered covered, and their demand is included in the optimization model.

$$\sum_{k} SC_{klt} \leq 1, \qquad \forall l, \forall t. \tag{32}$$

Constraint (32) guarantees that each retailer is covered by at most one integrated warehouse–collection center.

$$\sum_{k} Z_{klct} + UD_{lct} \geq \sum_{k} SC_{klt} . D_{lct}, \qquad \forall l, \forall c, \forall t. \tag{33}$$

Constraint (33) ensures that the demand of each covered retailer is satisfied, as much as possible, by products available in warehouses or produced in the current period. Any unmet demand is accounted for as lost sales in the model.

$$Z_{kct} \leq SC_{klt} \cdot D_{lct}, \qquad \forall k, \forall l, \forall c, \forall t. \tag{34}$$

Constraint (34) specifies that a retailer located within the warehouse coverage radius cannot receive more than its required demand in each period, while retailers outside the coverage area receive no shipments.

$$RZ_{lkct} = r1_c . Z_{klc,t-3}, \qquad \forall l, \forall k, \forall c, t \geq 4. \tag{35}$$

Constraint (35) defines the quantity of returned products from each retailer to each warehouse in each period. These returned products are proportional to the products shipped to retailers three periods earlier, according to a specified return rate.

$$RZ_{lkct} = 0, \qquad \forall l, \forall k, \forall c, t = 1,2,3. \tag{36}$$

Constraint (36) indicates that no returns from retailers to warehouses occur during the first three periods.

$$\sum_{l} RZ_{lkct} = \sum_{j} RY_{kjct}, \qquad \forall k, \forall c, \forall t. \tag{37}$$

Constraint (37) states that products returned from retailers to integrated warehouse–collection centers are subsequently transferred directly from warehouses to manufacturing centers.

$$\sum_{m} RD_{jmct} = (1 - r2_{c})\left(\sum_{k} RYT_{kjct} + \sum_{k} RY_{kjct}\right) + r3c\sum_{L,K} RZ_{lkct}, \qquad \forall j, \forall c, \forall t. \tag{38}$$

Eq. (38) shows that the quantity of products sent from manufacturing centers to disposal centers equals (1 − r2_c) of the total returned products received from warehouses and retailers.

$$\sum_{n} RR_{jnct} = (r2_{c})\left(\sum_{k} RYT_{kjct} + \sum_{k} RY_{kjct}\right) + (1 - r3c)\sum_{L,K} RZ_{lkct}, \qquad \forall j, \forall c, \forall t. \tag{39}$$

$$\sum_{n} RR_{jnct} = (r2_{c})\left(\sum_{k} RYT_{kjct} + \sum_{k} RY_{kjct}\right) + (1 - r3c)\sum_{L,K} RZ_{lkct}, \qquad \forall j, \forall c, \forall t. \tag{40}$$

Eqs. (39) and *(40)* represent the allocation of returned products to recycling centers versus manufacturing centers. A portion of returned products is directed to recycling centers for reuse, while the remaining portion is sent to disposal centers.

$$X_{ijrt}, Y_{jkct}, Z_{klct}, RZ_{lkct}, RYT_{kjct}, RY_{kjct}, RR_{jnct}, RD_{jmct}, SP_{rjt}, UD_{lct}, S_{kcc't} \geq 0, \quad \forall i, j, r, t, k, c, l, n, m. \tag{41}$$

$$Y_{jkct}, Z_{klct}, RZ_{lkct}, RYT_{kjct}, RY_{kjct}, RR_{jnct}, RD_{jmct}, UD_{lct}, S_{kct't} \in int, \qquad \forall j, t, k, c, l, n, m. \tag{42}$$

$$SC_{klt}, SO_{ijtt} \in \{0,1\}, \tag{43}$$

3 | Model Solution

In the model solution section, *Table 2* and *Fig. 3* present the objective function values (profit) for different problem instances under deterministic scenarios and various solution approaches. The results indicate that as the problem size increases, both the computational time and the obtained profit increase. However, this increase is not strictly linear. The growth in profit shows a nonlinear relationship with problem size due to variations in problem structure and scenario conditions [13–15].

Table 4. Comparison of objective function values (profit) across different problem instances for the base model and the proposed study under deterministic conditions.

Problem No	Objective Function Value (Deterministic)	Base Model | Solution Time (s) – Base Model	Objective Function Value (Deterministic)	Proposed Model | Solution Time (s) – Proposed Model
1	613003785.9	1.6416	600984103.78	2.052
2	1764683026	2.1496	1713284491.027	2.687
3	3375378125	1.2576	3277066140.89	1.572
4	2810002607	0.0368	2754904516.41	0.046
5	4783405661	1.28	4689613393.57	1.60
6	3968750745	1.232	3853156063.54	1.54
7	5873484929	1.16	5702412552.65	1.45
8	6058735681	1.144	5939936942.43	1.43
9	7104261256	1.6872	6897341025.35	2.109
10	6752062067	1.696	6619668692.88	2.120
11	7259487579	1.6992	7048046193.08	2.124
12	8932828674	3.544	8672649198.30	4.43

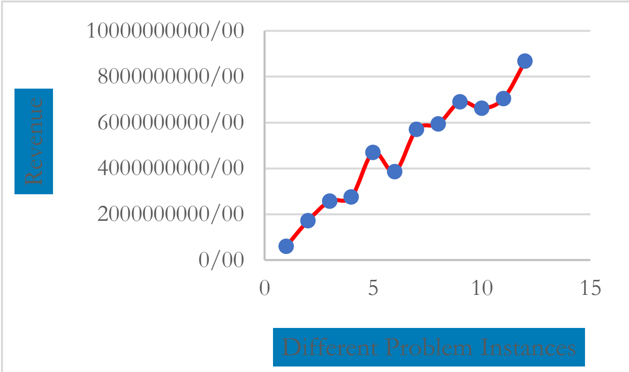

Fig. 3. Changes in the objective function value for different problem instances.

The comparison of objective function values between the base case (without considering the reuse of recovered products) and the proposed model (with consideration of product reuse) shows that the base case yields higher revenue than the proposed model. This behavior is illustrated in *Fig. 4*.

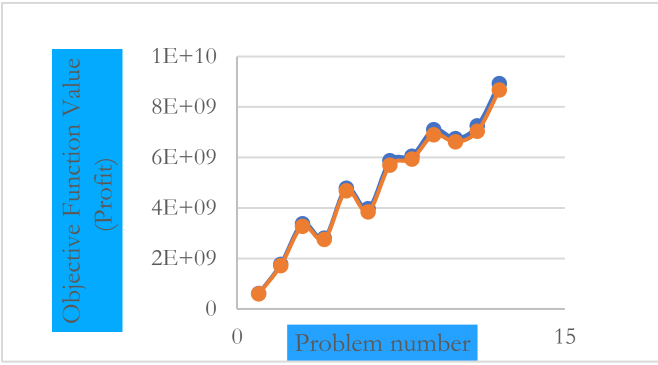

Fig. 4. Comparison of objective function values in the base case and the proposed study.

As illustrated in *Fig. 4*, there is a very small difference between the two cases in small-scale instances; however, as the problem size increases, the gap between the objective function values in the base case and the proposed model becomes larger. This trend is because the reuse of recovered products is not always economically beneficial, and in some industries, due to raw material scarcity and environmental considerations, product reuse is taken into account.

3.1 | Results

The comparison of objective function values between the base case (without considering product reuse) and the proposed model (with consideration of product reuse) shows that higher revenue is achieved in the base case. In small-scale instances, the difference between the two cases is negligible; however, as the problem size increases, the gap between their objective function values becomes more significant. The widening gap is due to the fact that product reuse is not always cost-effective, although in some industries it is necessary due to raw material shortages and environmental concerns. Another important result concerns the output of different scenario-based solution methods. The relationship indicating that the WS objective value is greater than the HN objective value has been examined. In addition, the Value of Stochastic Solution (VSS) and Expected Value of Perfect Information (EVPI) indicators were computed to evaluate stochastic solutions. These indicators reflect the value of considering uncertainty in the model. The comparison shows that only in problems 4, 8, and 12 do these measures not provide significant savings. Another key result is the comparison of two uncertainty modeling approaches used in this study: the scenario-based stochastic programming approach and the Malloy approach. Since in the Malloy approach all scenarios are solved simultaneously, the here-and-now approach was used for comparison purposes. The results indicate that for both small- and large-scale instances, there is a relatively small difference between the objective function values of the two methods [16], [17].

Based on the obtained results, the following managerial implications can be proposed:

Management should make every effort to reduce product returns, since although returned products can be partially recovered and reused as input for suppliers, they reduce the overall profit of the supply chain. Reuse of recovered products is applicable in industries such as automotive batteries; however, it is not feasible for all products. Therefore, managers should carefully evaluate the applicability of product reuse in their specific systems. Uncertainty modeling approaches significantly help in understanding different market conditions, enabling managers to better estimate the potential profit of the supply chain under various scenarios. Sensitivity analysis shows that as the problem size increases, the changes tend to become more linear, allowing managers to obtain more predictable profit estimations for larger supply chain systems. The reuse rate of recovered products can also be considered a key factor in supply planning decisions [18].

4 | Conclusion and Future Research Directions

To improve the results of this study, the following extensions are suggested:

I. Application of exact solution algorithms such as Benders decomposition and Frank–Wolfe methods to reduce computational time for large-scale instances.

II. Incorporating a time-dependent reuse rate to better represent real-world conditions.

III. Extending the model to include transportation facilities with different characteristics and heterogeneity.

4.1 | Summary

As demonstrated in this chapter, the following key conclusions were obtained:

I. The comparison between the base case (without product reuse) and the proposed model shows that the base case is more profitable across different problem sizes; however, due to resource scarcity in the production of certain products, product reuse becomes inevitable. Therefore, this necessity should be considered in future mathematical models.

II. The use of scenario-based stochastic programming and Malloy's approach leads to cost savings. Out of 12 test problems, only 3 instances did not show significant savings. Therefore, with approximately 70% confidence, these approaches can be considered beneficial [19].

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All research activities and manuscript development were conducted by the author, who also approved the final manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflict of Interest

The author declares that in this paper, there is no conflict of interests.

Consent for Publication

The author has provided their consent for the publication of this manuscript.

References

[1] Eskandarpour, M., Dejax, P., Miemczyk, J., & Péton, O. (2015). Sustainable supply chain network design: An optimization-oriented review. *Omega, 54*, 11–32. https://doi.org/10.1016/j.omega.2015.01.006

[2] Bortolini, M., Faccio, M., Ferrari, E., Gamberi, M., & Pilati, F. (2016). Fresh food sustainable distribution: cost, delivery time and carbon footprint three-objective optimization. *Journal of food engineering, 174*, 56–67. https://doi.org/10.1016/j.jfoodeng.2015.11.014

[3] Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations research, 43*(2), 264–281. https://doi.org/10.1287/opre.43.2.264

[4] Chopra, S., & Meindl, P. (2019). Supply chain management. Strategy, planning & operation. In *Das summa summarum des management: die 25 wichtigsten werke für strategie, führung und veränderung* (pp. 265–275). Springer. https://doi.org/10.1007/978-3-8349-9320-5_22

[5] Ahumada, O., & Villalobos, J. R. (2009). Application of planning models in the agri-food supply chain: A review. *European journal of operational research, 196*(1), 1–20. https://doi.org/10.1016/j.ejor.2008.02.014

[6] ISO. (2011). *Environmental management-material flow cost accounting-general framework*. https://www.iso.org/standard/50986.html

[7] Nahmias, S. (1982). Perishable inventory theory: A review. *Operations research, 30*(4), 680–708. https://doi.org/10.1287/opre.30.4.680

[8] Govindan, K., Soleimani, H., & Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European journal of operational research, 240*(3), 603–626. https://doi.org/10.1016/j.ejor.2014.07.012

[9] Melkote, S., & Daskin, M. S. (2001). Capacitated facility location/network design problems. *European journal of operational research, 129*(3), 481–495. https://doi.org/10.1016/S0377-2217(99)00464-6

[10] Leung, S. C. H., Tsang, S. O. S., Ng, W.-L., & Wu, Y. (2007). A robust optimization model for multi-site production planning problem in an uncertain environment. *European journal of operational research, 181*(1), 224–238. https://doi.org/10.1016/j.ejor.2006.06.011

[11] Lee, H. L., & Billington, C. (1993). Material management in decentralized supply chains. *Operations research, 41*(5), 835–847. https://doi.org/10.1287/opre.41.5.835

[12] Alshamsi, A., & Diabat, A. (2015). A reverse logistics network design. *Journal of manufacturing systems, 37*, 589–598. https://doi.org/10.1016/j.jmsy.2015.02.006

[13] Wolsey, L. (1998). *Integer Programming*. Wiley-Interscience. https://books.google.com/books?hl=en&lr=&id=knH8DwAAQBAJ&oi=fnd&pg

[14] Bertsimas, D., & Tsitsiklis, J. N. (1997). I*ntroduction to linear optimization* (Vol. 6). Athena scientific Belmont, MA. https://www.researchgate.net/publication/235558951_Introduction_to_Linear_Optimization

[15] Stadtler, H., Stadtler, H., Kilger, C., Kilger, C., Meyr, H., & Meyr, H. (2015). *Supply chain management and advanced planning: Concepts, models, software, and case studies*. Springer. https://doi.org/10.1007/978-3-540-74512-9

[16] Thierry, M., Salomon, M., Van Nunen, J., & Van Wassenhove, L. (1995). Strategic issues in product recovery management. *California management review, 37*(2), 114–136. https://doi.org/10.2307/41165792

[17] Birge, J. R. (1982). The value of the stochastic solution in stochastic linear programs with fixed recourse. *Mathematical programming, 24*(1), 314–325. https://doi.org/10.1007/BF01585113%0A%0A

[18] Fleischmann, M., Bloemhof-Ruwaard, J. M., Dekker, R., der Laan, E., Van Nunen, J. A. E. E., & Van Wassenhove, L. N. (1997). Quantitative models for reverse logistics: A review. *European journal of operational research, 103*(1), 1–17. https://doi.org/10.1016/S0377-2217(97)00230-0

[19] Geoffrion, A. M. (1972). Generalized benders decomposition. *Journal of optimization theory and applications, 10*(4), 237–260. https://doi.org/10.1007/BF00934810%0A%0A