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A Clustering-Based Decomposition Framework for Solving the Feeder Vehicle Routing Problem

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Abstract

The rapid expansion of e-commerce has made home delivery an essential service for both customers and businesses. However, parcel delivery operations are increasingly challenged by traffic restrictions, high operational costs, and limited resource availability. A promising strategy for addressing these challenges is the simultaneous use of a heterogeneous fleet, comprising trucks and motorcycles, in distribution operations. Despite its potential advantages, the planning of such hybrid delivery systems is highly complex, as their structural characteristics and operational constraints give rise to large-scale optimization models that are difficult to solve within a reasonable computational time. To overcome this limitation, this study proposes a matheuristic approach for the problem under consideration. The proposed method first decomposes the original problem into smaller subproblems using clustering techniques. Each subproblem is then solved by means of a mathematical optimization model, and the resulting solutions are integrated to construct the final delivery plan. Computational experiments on a set of standard benchmark instances demonstrate that the proposed strategy achieves an average reduction of 32.51% in the objective function value compared with traditional delivery methods.

Keywords: Feeder VRP, Clustering, Truck and motorcycle, Matheuristic, Optimization, Routing.

1 | Introduction

With the growth of e-commerce and the persistence of changes in consumer purchasing behavior, particularly the increasing preference for online shopping, the demand for delivery services has continued to expand across societies and business sectors. As a result, providing delivery services that are both fast and cost-effective has become a critical priority for modern enterprises. The final stage of the distribution process, in which parcels are transported to customers' residences, is commonly referred to as last-mile delivery. This stage is widely recognized in the literature as the most costly, least efficient, and most environmentally burdensome segment of the supply chain [1].

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Key challenges in last-mile delivery operations include high demand volumes, complex transportation planning, failed delivery attempts, limited parking availability, restricted access to certain areas, adverse weather conditions, rising fuel costs, severe traffic congestion, and greenhouse gas emissions from delivery vehicles [2].

One approach to mitigating these challenges is to use smaller and more agile vehicles, such as motorcycles, for parcel delivery. Motorcycles demonstrate significantly better performance than trucks in congested streets, inaccessible areas, and locations lacking adequate parking. However, these vehicles are constrained by limited cargo capacity. If they conduct delivery operations independently, they require frequent returns to the depot, which reduces operational efficiency. Therefore, motorcycles can be deployed alongside a larger vehicle, such as a truck, which serves as a mobile warehouse (feeder).

This collaboration helps overcome the capacity limitations of smaller vehicles while leveraging their maneuverability advantages. This novel delivery method is known in the literature as the Feeder Vehicle Routing Problem (FVRP), first proposed by Tu et al. [3].

The FVRP consists of a central depot, a set of customers, and a heterogeneous fleet of vehicles dispatched from the depot. The fleet includes two vehicle classes: large vehicles, such as delivery trucks, and small vehicles, such as motorcycles or bicycles. Small vehicles typically have substantially lower operating costs than large vehicles, can navigate congested urban areas more efficiently, and are generally associated with lower emissions. In contrast, large vehicles provide greater carrying capacity and can travel longer distances without frequent refueling, although they incur higher overall operating costs [4].

Crucially, both small and large vehicles must conduct the delivery operations synchronously and collaboratively to maximize the benefits of both vehicle types. In other words, the small and large vehicles may rendezvous at a shared location, termed a joint node. At this location, the smaller vehicle is reloaded by the larger vehicle up to its capacity [5].

Based on the stated assumptions, the FVRP is an extension of the Vehicle Routing Problem (VRP), which belongs to the NP-Hard class of optimization problems [6]. The coordination and synchronization requirements between the two vehicle types further increase the complexity of planning and solving the FVRP. Consequently, obtaining optimal solutions for real-world instances within a reasonable computational time is often impractical, highlighting the need for alternative solution approaches capable of producing high-quality solutions. Previous studies have proposed various heuristic and metaheuristic algorithms for this class of problems [7]. However, to the best of our knowledge, a mathheuristic approach has not yet been applied to the FVRP. In this type of solution method, the original problem is first decomposed into smaller, well-structured Subproblems (SPs) that can be solved to optimality within a short computational time. Each SP is then solved using optimization software, and the final solution is constructed by integrating the solutions of the SPs. The main advantage of matheuristic methods over purely heuristic and metaheuristic approaches is that they exploit exact optimization techniques at the SP level, which can lead to higher-quality solutions that are closer to the global optimum [8].

In the current research, a Decomposition-based Matheuristic (DMH) framework is proposed for the FVRP for the first time, and it will be referred to as the DMH-FVRP framework throughout this paper. Another contribution of this study is the development of a novel heuristic approach for eliminating subtours in the solution process of the classical VRP. Instead of incorporating all Subtour Elimination Constraints (SECs) into the model at once, the proposed method adds only those constraints required to eliminate the subtours identified in the previous solution. This iterative procedure reduces the total number of SECs ultimately included in the model and, consequently, decreases the computational time required to solve the VRP instance.

The remainder of the article is structured as follows. Section 2 reviews the literature pertinent to this research. Section 3 defines the assumptions of the problem under study, and Section 4 describes the developed solution framework, and finally, the concluding section presents the computational results and the final summary of the research.

2 | Literature Review

The FVRP is an extension of the classical VRP, a widely studied combinatorial optimization problem first introduced by Dantzig and Ramser [9]. In the VRP, a set of vehicle routes is planned with the objective of minimizing the total time or distance traveled to deliver goods to customers. This process constructs one tour for each vehicle, ensuring that every customer is visited exactly once [2].

Researchers have gradually incorporated new assumptions to bring the problem closer to real-world conditions. These include vehicle capacity constraints, distribution from multiple depots, the inclusion of time windows for customer visits, vehicle heterogeneity, conditions of uncertainty regarding customer demand or route lengths, and the consideration of environmental factors [10].

Tu et al. first introduced the concept of the Line-Haul Feeder Vehicle Routing Problem (LFVRP) with a predetermined virtual depot for food delivery in Taiwan. This strategy employs two types of vehicles to serve two distinct categories of customers. The first category includes customers with sufficient parking space at the service location, allowing either vehicle type to be used for service. The second category consists of customers lacking adequate parking, meaning only smaller vehicles can be used to serve them. When a small vehicle is empty, it may deviate from its designated route to reload from the larger vehicle [3].

One assumption that reduced the flexibility of the initial solutions to this problem was the predetermination of joint nodes (virtual depots) where the two vehicle types could rendezvous. Chen et al. [4] addressed this limitation by introducing the Line-Haul Feeder VRP with Virtual Depot (LFVRP-VD) as an extension of the original problem type. Similarly, Chen et al. investigated the LFVRP with Virtual Depot and Time Windows (LFVRP-VDTW) [5].

Brandstätter and Reimann [11] studied a new version of the LFVRP, known as the VRP with synchronization constraints, where two types of vehicles serve two different types of customers.

Various methods, predominantly based on heuristic algorithms, have been used to solve these problems. In this domain, Chen et al. proposed two heuristic methods, referred to as the cost-sharing method and the threshold method, for the LFVRP-VD model to minimize travel and vehicle waiting costs [4]. Additionally, Chen et al. utilized a two-stage heuristic algorithm, which includes tabu search and a sequential insertion method, to solve the LFVRPTW problem [5].

3 | Problem Description

This problem considers a parcel delivery system consisting of one delivery truck and one motorcycle with a limited carrying capacity. Both vehicles are required to depart from a central depot and return to it after all customers have been served. A key constraint is that each customer must be visited exactly once.

Due to the limited cargo capacity of the motorcycle, frequent returns to the central depot may reduce operational efficiency. To address this limitation, the motorcycle is allowed to rendezvous with the truck at selected points along the route for reloading. These meeting points, referred to as joint nodes, are not predetermined; rather, any customer node may potentially serve as a joint node. The distances between the nodes are considered input parameters of the problem.

The objective of the problem is to minimize the total time required to serve all customers. The remaining operational assumptions are as follows:

I. Work shift: A maximum eight-hour work shift is enforced.

II. Demand coverage: It is assumed that customer demand within each work shift is fully satisfied by the truck and motorcycle, ensuring no customer demand remains unfulfilled.

III. Time parameters: The model incorporates the time required for the motorcycle to reload from the truck at the joint nodes, the service time provided by each vehicle at the customer location, and the respective capacities of the truck and the motorcycle.

The modeling framework employed in this research is based on the model proposed by Salehi and Behnamian [12], with modifications implemented to align with the specific assumptions of the current study. This model is formulated as a mixed integer programming model, the full details of which are provided in the Appendix of this paper. While the objective function in the original article was to minimize fixed and variable operating costs, the objective function in this research is defined as the minimization of the total delivery operation time.

4 | Solution Algorithm

This section details the DMH-FVRP solution framework designed for solving the FVRP, along with an explanation of the heuristic subtour elimination method.

4.1 | The DMH-FVRP Framework for Solving the FVRP

This research proposes a DMH framework for solving the FVRP. The core idea of this framework is to decompose the primary large-scale problem into several smaller SPs using a clustering algorithm and then solve each SP exactly and independently. In the initial step, the set of customer nodes is partitioned into multiple clusters using a hierarchical clustering method. Unlike conventional clustering approaches, this method imposes an explicit constraint on the number of nodes in each cluster to control the computational time required to solve the resulting SPs. Specifically, the maximum cluster size is set to 15 nodes, based on experimental results indicating that the exact solution for SPs of this size can be achieved in less than one minute. As cluster size increases, the solution time rises significantly, reducing the time efficiency of the overall algorithm. Following the clustering phase, each SP is solved independently using an exact solver to determine its optimal solution. Finally, the solutions obtained from the SPs are interconnected through an integration process to generate the final answer for the primary problem.

The proposed approach leverages the advantages of both exact and heuristic algorithms. On one hand, the use of heuristic methods effectively reduces the overall solution time, while on the other, the quality of the solution is nearly guaranteed by solving the SPs exactly. This framework is particularly efficient and effective for large-scale problems that cannot be solved using traditional exact methods within a reasonable timeframe.

4.2 | Heuristic SP Integration Algorithm

To integrate the SP solutions, the heuristic employed in this research is the cheapest insertion algorithm proposed by Renaud et al. [13] for determining the sequence of cluster visits. In this method, a representative point is first selected for each cluster. In the present study, this point is defined as the customer node closest to the cluster centroid. The algorithm starts from an initial cluster and then sequentially inserts the remaining clusters into the route, thereby determining the order in which the clusters are visited. At each step, the next cluster is selected based on the smallest increase it causes in the total route length. Once the final cluster sequence is obtained, the optimal routes generated for the individual SPs are connected according to this sequence. Owing to its low computational cost, structured procedure, and short execution time, this method provides an efficient mechanism for integrating partial solutions into a complete final solution.

4.3 | Heuristic Subtour Elimination Method

Solving a VRP can be computationally demanding due to the large number of SECs. Therefore, this study proposes a novel iterative algorithm for solving VRP instances based on Selective Subtour Elimination Constraints (SSECs).

The main concept is to avoid the significant computational complexity that arises from introducing all SECs at the beginning. Instead, only the necessary constraints required at each step are added to the model.

Therefore, the model is initially solved without considering any SECs. In the next step, all existing subtours in this solution are identified, and only the constraints corresponding to the elimination of those specific subtours are added to the model. The problem is then resolved with the updated model. This iteration persists until the resulting solution provides a complete tour without any subtours.

This approach significantly reduces the number of constraints in the problem, leading to increased computational efficiency and using less time and memory compared to adding all SECs at once. A comparison of the computational results between the proposed iterative approach and the method of adding all SECs simultaneously will be presented in the numerical results section.

5 | Computational Results

5.1 | DMH-FVRP Framework Implementation Results on a Small Example

To better illustrate the functionality of this framework, an example is provided below. As shown in *Fig. 1.a*, this example consists of a set of 28 customers symmetrically dispersed around the central depot and grouped into four clusters. Due to the high number of customers, obtaining an exact solution for this problem in a logical time frame is not feasible.

Hence, the framework developed in this research initially decomposes the problem into four SPs, as shown in *Fig. 1.b*, each of which can be solved exactly in a short time.

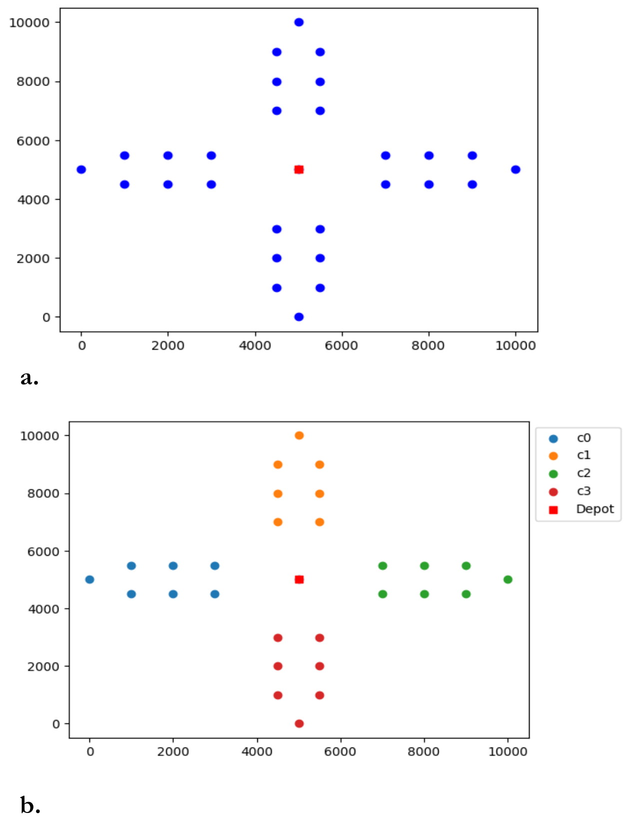

Fig. 1. a. Customer distribution in sample problem, and b. Customer distribution after clustering.

Accordingly, the truck and motorcycle simultaneously enter a cluster, and after servicing the customers within that cluster, they meet to allow the motorcycle to be reloaded by the truck. The delivery process then continues in the next cluster until all customers have been serviced, and both vehicles finally return to the central depot. The final solution for the primary problem is thus obtained, as shown in *Fig. 2*. The interpretation of the final solution is as follows: The truck and motorcycle start their route from the central depot. The truck services customers 5, 8, and 7 in sequence. Simultaneously, the motorcycle services customers 3, 4, 2, and 6. They then rendezvous at customer node 9, where the motorcycle receives four new parcels for delivery to customers 10, 11, 15, and 14, and they continue their routes. This process persists until all customers are serviced, and both vehicles ultimately return to the depot.

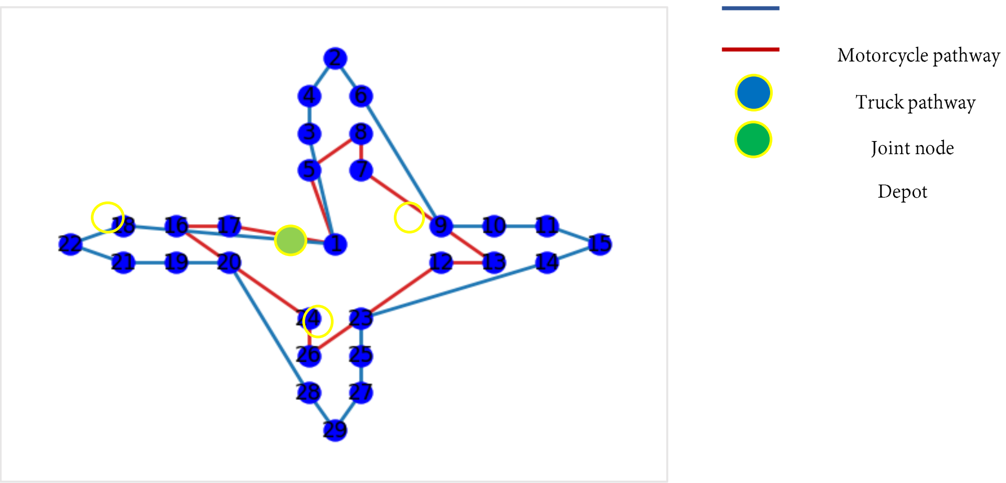

Fig. 2. Ultimate routing solution of the sample problem, obtained by the DMH-FVRP framework.

5.2 | Implementation Results of the Heuristic SSEC Method on a Small Example

To better illustrate the functionality of the developed algorithm, its application to a small-scale example is described here. As shown in *Fig. 3*, a sample problem consisting of 15 customer nodes was considered, and the SSEC algorithm was used for routing parcel delivery to these customers. Using this approach, constraints that eliminate the subtours generated in the preceding solution step are iteratively added to the problem, thereby preventing the recurrence of those specific subtours.

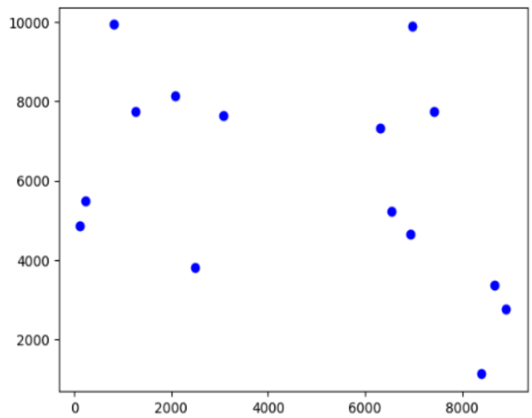

Fig. 3. Distribution of customers in the sample problem.

By adopting this method, the algorithm successfully reached the optimal solution in only three stages (as shown in *Fig. 4*) by adding just 118 constraints to the model, and doing so in 29.13 seconds. In contrast, achieving the same result when all SCEs (16,383) are included in the problem from the outset requires 371.19 seconds. These results confirm that the proposed method significantly reduces problem complexity and, consequently, the solution time, without generating the entire set of SECs.

5.3 | Implementation Results on Standard Test Problems

This section presents the results of implementing the DMH-FVRP framework on standard sample problems derived from the OR-Library repository[1]. Specifically, the study utilizes the standard problems designed by Solomon in 1987, found within the vehicle routing section, using the VRP_06 dataset.

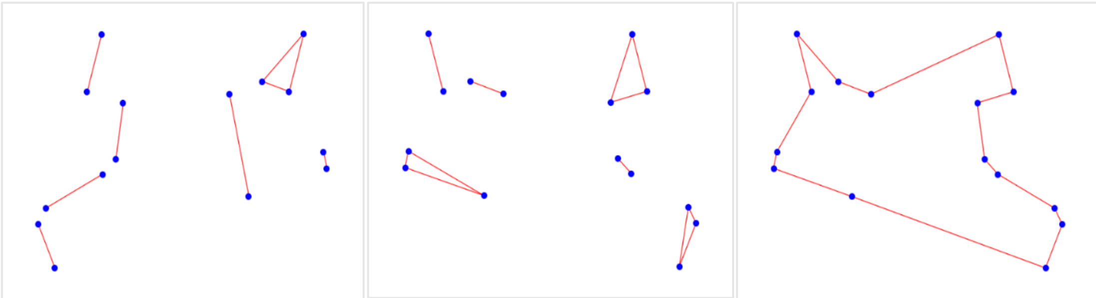

Fig. 4. Stages of the SSEC algorithm until obtaining the optimal solution.

The 25 test problems examined range in size from 5 to 50 customers, all of whom must be serviced within an eight-hour work shift by a single motorcycle and a single truck acting as a feeder vehicle, after which both vehicles must return to the central depot. The distances between nodes are calculated using Euclidean distances.

The parameters for motorcycle capacity, service time, and reloading time are assumed to follow a uniform distribution in the ranges of [3-6], [1-4], and [2-8], respectively. The proposed algorithms were coded using the Python programming language, and the exact solution of the SPs was obtained by the CPLEX solver. The hardware used in these calculations includes 20 GB of DDR4 RAM and an Intel Core i5-8850U 2.3GHz processor. The results of these computations are summarized in *Table 1*.

Table 1. Computational results on the standard test instances.

	FVRP				VRP					
	DMH-FVRP		Exact Solution (CPLEX)		SSEC		Exact Solution (CPLEX)			
Number of Customers	Time(s)	Objective	Time(s)	Objective	Time(s)	Objective	Time(s)	Objective	Gap between VRP & FVRP (%)	Gap with Exact Solution (%)
5	0.26	4135	0.15	4135	0.1	5523	0.09	5523	14.67	0
7	2.13	7803	1.64	7803	0.43	11056	0.65	11056	11.03	0
9	9.71	10204	13.19	9404	2.61	14245	8.92	14245	10.34	8.51
11	10.54	12856	29.14	11361	6.22	16154	18.16	16154	12.19	9.25
13	12.36	13421	45.84	12751	9.53	21403	27.87	21403	16.85	2.17
15	15.59	17052	59.42	14899	13.24	24056	46.33	24056	18.86	7.82
17	17.23	18438	380.26	16916	16.95	31451	210.15	31451	21.04	7.75
19	19.25	20431	890.43	19093	19.14	35421	521.61	35421	17.35	5.82
21	22.74	23560	-	-	21.06	38651	820.60	38651	24.79	-
23	26.54	24778	-	-	25.64	40895	-	-	29.22	-
25	28.08	26646	-	-	27.35	42956	-	-	31.08	-
27	30.47	30062	-	-	29.01	44626	-	-	28.36	-
29	31.20	30749	-	-	30.26	46306	-	-	29.74	-
31	32.17	34799	-	-	31.47	49877	-	-	33.25	-
33	33.87	33805	-	-	32.39	50642	-	-	33.85	-
35	34.26	39249	-	-	33.22	55981	-	-	34.12	-
37	34.95	40726	-	-	34.89	58403	-	-	35.98	-
39	36.11	41737	-	-	35.62	60423	-	-	38.62	-
41	37.84	43714	-	-	36.26	63159	-	-	39.09	-
43	39.87	46486	-	-	38.52	67485	-	-	42.84	-
45	40.45	49699	-	-	39.76	69123	-	-	40.94	-
47	48.13	49014	-	-	43.05	71410	-	-	43.45	-
49	52.47	53451	-	-	49.15	78512	-	-	41.62	-
51	55.59	55808	-	-	51.95	82451	-	-	38.24	-
53	58.64	58411	-	-	52.12	85941	-	-	39.79	-
Average	29.21	31481	-	-	27.19	46646	-	-	32.51%	-

[1] https://people.brunel.ac.uk/~mastjjb/jeb/info.html

Table 1 presents the computational results for instances with different numbers of customers. The first column reports the number of customers. Columns two to five provide the objective function values and solution times for the FVRP scenario, obtained using the DMH-FVRP algorithm and the exact algorithm. Similarly, columns six to nine present the objective function values and solution times for the VRP scenario, obtained using the SSEC algorithm and the exact algorithm. The tenth column reports the difference between the VRP and FVRP solutions, while the final column shows the percentage deviation of the DMH-FVRP solution from the optimal FVRP solution.

As observed across all sample problems, the objective function value for the traditional VRP delivery plan is higher than that of the FVRP delivery plan. The lower objective function value of the FVRP delivery plan is because the motorcycle has a higher speed compared to the truck, allowing it to traverse the same distance in a shorter time. It is also noted that in FVRP instances with fewer than 15 customers, the solutions obtained by the exact algorithm and the DMH-FVRP framework are identical, as the clustering process places all customers into a single cluster, thus yielding the same result for both problems. However, as the number of customers increases and clustering becomes necessary, the solution deviates from the global optimum.

Furthermore, the computation time for the VRP instances is generally shorter compared to the FVRP, which is justifiable given the higher complexity of the FVRP problem and the constraints associated with the synchronization of the two vehicle types.

The results indicate that the SSEC method makes it possible to obtain the optimal VRP solution for medium-sized problems in a short time, whereas calculating this response using the exact solver is only feasible for small-scale problems. Furthermore, based on the exact solutions computed for the small-sized FVRP instances, an average difference of 8.16% is observed between the exact solution and the solution derived from the DMH-FVRP framework. This value confirms the acceptable performance of the developed algorithm in solving the FVRP.

Finally, the comparison between the VRP and FVRP solutions shows an average improvement of 32.51% in the objective function value, which demonstrates the superiority of the synchronized truck-and-motorcycle delivery strategy over the traditional method of parcel delivery. This substantial improvement is achieved while the DMH-FVRP framework requires only 5.62% more computational time to obtain the FVRP solution than is required for the VRP solution.

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Authors' Contributions

The author solely conducted the research and prepared the manuscript and has approved its final version.

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Conflict of Interest

There are no competing interests to declare.

Data Availability

The data are available from the corresponding author upon reasonable request.

References

[1] Kiba-Janiak, M., Marcinkowski, J., Jagoda, A., & Skowrońska, A. (2021). Sustainable last mile delivery on e-commerce market in cities from the perspective of various stakeholders. Literature review. *Sustainable cities and society*, *71*, 102984. https://doi.org/10.1016/J.SCS.2021.102984

[2] Simoni, L., & Laura, B. (2020). Effects of social, environmental, and institutional factors on sustainability report assurance: evidence from European countries. *Meditari accountancy research, 28*, 1059–1087. https://doi.org/10.1108/MEDAR-03-2019-0462

[3] Tu, s., S. Lai, & Y. Li. (2001). *Application of the vehicle routing problem with time windows — an example of lunch box delivery*.

[4] Chen, H. K., Chou, H. W., Hsueh, C. F., & Ho, T. Y. (2011). The linehaul-feeder vehicle routing problem with virtual depots. *IEEE transactions on automation science and engineering*, *8*(4), 694-704. https://doi.org/10.1109/TASE.2011.2142304

[5] Chen, H. K. (2015). Issues for the linehaul-feeder vehicle routing problem with virtual depots and time windows. *Journal of the Eastern Asia society for transportation studies*, *11*, 678-692. https://doi.org/10.11175/easts.11.678

[6] Li, X., Leung, S. C., & Tian, P. (2012). A multistart adaptive memory-based tabu search algorithm for the heterogeneous fixed fleet open vehicle routing problem. *Expert systems with applications*, *39*(1), 365-374. https://doi.org/10.1016/j.eswa.2011.07.025

[7] Sarbijan, M. S., & Behnamian, J. (2022). Multi-fleet feeder vehicle routing problem using hybrid metaheuristic. *Computers & operations research*, *141*, 105696. https://doi.org/10.1016/j.cor.2022.105696

[8] Boschetti, M. A., & Maniezzo, V. (2022). Matheuristics: using mathematics for heuristic design. *4or*, *20*(2), 173-208. https://doi.org/10.1007/S10288-022-00510-8

[9] Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management science*, *6*(1), 80-91. https://doi.org/10.1287/MNSC.6.1.80

[10] Wang, Y., Wang, L., Chen, G., Cai, Z., Zhou, Y., & Xing, L. (2020). An improved ant colony optimization algorithm to the periodic vehicle routing problem with time window and service choice. *Swarm and evolutionary computation, 55*, 100675. https://doi.org/10.1016/j.swevo.2020.100675

[11] Brandstätter, C., & Reimann, M. (2018). The Line-haul Feeder Vehicle Routing Problem: Mathematical model formulation and heuristic approaches. *European journal of operational research*, *270*(1), 157–170. https://doi.org/10.1016/j.ejor.2018.03.014

[12] Salehi Sarbijan, M., & Behnamian, J. (2022). Multi-fleet feeder vehicle routing problem using hybrid metaheuristic. *Computers & operations research*, *141*, 105696. https://doi.org/10.1016/J.COR.2022.105696

[13] Renaud, J., Boctor, F. F., & Ouenniche, J. (2000). A heuristic for the pickup and delivery traveling salesman problem. *Computers & operations research*, *27*(9), 905–916. https://doi.org/10.1016/S0305-0548(99)00066-0

Appendix

The mixed integer programming model used in this research is as follows [7]:

Sets

I: Set of all customer nodes.

N: Set of all customer nodes including the depot.

Indices

(i,j): Indices corresponding to nodes.

c: Index for the truck.

m: Index for the motorcycle.

Parameters

T_{ij}^{c}: Truck travel time between nodes i and j.

S_{i}^{c}: Service time for the customer i by the truck.

Q^{c}: Truck capacity.

T: Maximum time of one work shift.

M: A large number.

T_{ij}^{m}: Motorcycle travel time between nodes i and j.

S_{i}^{m}: Service time for the customer i by the motorcycle.

Q^{m}: Motorcycle capacity.

cc: Motorcycle reloading time.

Decision variables

x_{ij}: Binary variable, equals 1 if edge ij is traveled by the truck, 0 otherwise.

w_i: Binary variable, equals 1 if node i is a joint node, 0 otherwise.

f_i: Motorcycle inventory level before visiting the customer i.

b_i: Motorcycle cumulative working time before visiting the customer i.

y_{ij}: Binary variable, equals 1 if edge ij is traveled by the motorcycle, 0 otherwise.

$w_{ij} = f_i * w_i * y_{ij}$: Quantity of inventory transferred from the truck to the motorcycle at the joint node i en route to the destination.

g_i: Truck inventory level before visiting the customer i.

a_i: Truck cumulative working time before visiting customer i.

$$\min \sum_{i \in N} \sum_{j \in N: i \neq j} (t_{ij}^{c} + s_{j}^{c})\, x_{ij} + \sum_{i \in N} \sum_{j \in N: i \neq j} (t_{ij}^{m} + s_{j}^{m})\, y_{ij} + cc \sum_{i \in N} w_i. \quad (1)$$

$$\sum_{j \in I: j \neq i} x_{ij} \leq 1 \; \forall\, i \in N. \quad (2)$$

$$\sum_{i\in N:i\neq l} x_{il} = \sum_{j\in N:l\neq j} x_{lj}, \quad \forall\, j \in N. \tag{3}$$

$$\sum_{j\in I:j\neq i} y_{ij} \leq 1, \quad \forall\, i \in N. \tag{4}$$

$$\sum_{i\in N:i\neq l} y_{il} = \sum_{j\in N:l\neq j} y_{lj}, \quad \forall\, j \in I. \tag{5}$$

$$\sum_{i\in N:i\neq j} x_{ij} \geq w_j, \quad \forall\, j \in N. \tag{6}$$

$$\sum_{i\in N:i\neq j} (x_{ij} + y_{ij}) \leq 1 + w_j, \quad \forall\, i \in N, j \in I : i \neq j. \tag{7}$$

$$g_i \leq Q^c, \quad \forall\, i \in N. \tag{8}$$

$$f_i \leq Q^m, \quad \forall\, i \in N. \tag{9}$$

$$f_j \geq f_i - (y_{ij} - w_i)q_i - Q^m(1 - y_{ij} + w_i + w_j), \quad \forall\, j \in N. \tag{10}$$

$$f_j \leq f_i - (y_{ij} - w_i)q_i + Q^m(1 - y_{ij} + w_i + w_j), \quad \forall\, j \in N. \tag{11}$$

$$f_j \geq q_j y_{ij} - Q^m w_j, \quad \forall\, j \in N. \tag{2}$$

$$f_j \leq Q^m(1 - w_j), \quad \forall\, j \in N. \tag{13}$$

$$\sum_{i\in N}\sum_{j\in N:i\neq j} q_j x_{ij} + \sum_{i\in N}\sum_{j\in N:i\neq j} q_j y_{ij} - \sum_{j\in N} q_j w_j = \sum_{j\in N} q_j, \quad \forall\, j \in N. \tag{14}$$

$$g_j \geq g_i - x_{ij}q_i - \sum_{l\in N} wyf_{il} - M(1 - x_{ij}), \quad \forall\, i \in N, j \in I : i \neq j. \tag{15}$$

$$g_j \leq g_i - x_{ij}q_i - \sum_{l\in N} wyf_{il} + M(1 - x_{ij}), \quad \forall\, i \in N, j \in I : i \neq j. \tag{16}$$

$$M\sum_{i\in N}(x_{ij} + y_{ij}) \geq q_j, \quad \forall\, i \in N, j \in I : i \neq j. \tag{17}$$

$$a_i \leq T, \quad \forall\, i \in N. \tag{18}$$

$$a_j \geq a_i + t_{ij}^c + s_i^c - 2T(1 - x_{ij}), \quad \forall\, j \in N. \tag{19}$$

$$b_i \leq T, \quad \forall\, i \in N. \tag{20}$$

$$b_j \geq b_i + t_{ij}^m + s_i^m + w_i(cc - s_i^m) - 2T(1 - y_{ij}), \quad \forall\, i \in N, j \in I : i \neq j. \tag{21}$$

$$b_j \geq a_j - 2T(1 - w_j), \quad \forall\, j \in N. \tag{22}$$

$$a_j \geq b_i + t_{ij}^c + s_i^c - 2T(1 - x_{ij}) - 2T(1 - w_i), \quad \forall\, i \in N, j \in I : i \neq j. \tag{23}$$

$$wyf_{ij} \geq f_j - Q^c(1 - w_i) - Q^c(1 - y_{ij}), \ \forall\, i \in N, j \in I : i \neq j. \tag{24}$$

$$wyf_{ij} \leq f_j - Q^c(1 - w_i) + Q^c(1 - y_{ij}), \quad \forall\, i \in N, j \in I : i \neq j. \tag{25}$$

$$wyf_{ij} \leq Q^m(w_i), \ \forall\, i \in N, j \in I : i \neq j. \tag{26}$$

$$wyf_{ij} \leq Q^m(y_{ij}), \quad \forall\, i \in N, j \in I : i \neq j. \tag{27}$$

$$wyf_{ij} \geq 0, \quad \forall\, i \in N, j \in I : i \neq j. \tag{28}$$

$$f_i, g_i, a_i, b_i \geq 0. \tag{29}$$

$$x_{ij}, y_{ij}, w_i \in \{0,1\}. \tag{30}$$