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# Simultaneous Optimization of Production, Maintenance, and Quality in a Multi-Product System under Demand Scenarios

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## Abstract

This study addresses the integrated optimization of production, Preventive Maintenance (PM), inspection, and quality control decisions in a multi-period and multi-product manufacturing system subject to two distinct types of stochastic failures. Type I failure represents a shift from an in-control to an out-of-control state, leading to the production of nonconforming items, which can only be identified through inspection. Type II failure is a complete system breakdown that halts production and requires corrective maintenance. The proposed model incorporates multiple levels of PM, each associated with different costs, durations, and age-reduction effects on the production system. To reflect real-world market variability, three demand scenarios are considered, representing high, medium, and low demand levels with different associated lost-order costs. The objective is to maximize the expected total profit by jointly determining production quantities, inspection schedules, PM levels, and inventory and sales decisions. The problem is formulated as a mixed-integer nonlinear programming model. Due to its complexity, a step-by-step solution procedure and a genetic algorithm are developed and compared with exact solutions obtained via GAMS for smaller instances. Computational results indicate that increasing the number of inspections up to a certain threshold improves profitability by reducing nonconforming output and failure-related losses, while the optimal PM level depends on the interaction between maintenance costs, production requirements, and demand conditions. The findings provide practical insights for managers in knowledge-intensive and high-mix manufacturing environments seeking to balance equipment reliability, product quality, and customer service under uncertain demand.

**Keywords:** Simultaneous optimization, Failure, Inspection, Preventive maintenance, Demand scenarios.

## 1 | Introduction

Modern manufacturing systems operate in an increasingly uncertain and competitive environment in which production efficiency, product quality, equipment reliability, and timely delivery must be managed simultaneously. Production equipment is subject to deterioration and stochastic failures, which can interrupt production, reduce available capacity, increase maintenance and operating costs, and adversely affect product

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quality. Consequently, maintenance is no longer regarded as an isolated technical activity; rather, maintenance decisions are closely interconnected with production planning, quality control, inventory management, and customer service decisions. Recent studies emphasize that jointly coordinating these decisions can provide substantial operational and economic benefits compared with treating production, maintenance, and quality as independent functions [1–3].

Preventive Maintenance (PM) is one of the principal strategies for controlling equipment deterioration and reducing the probability of unexpected failures. However, PM itself consumes time and financial resources and may temporarily reduce production capacity. Therefore, excessive maintenance may increase operating costs and cause unnecessary production interruptions, whereas insufficient maintenance can increase failure frequency, downtime, defective production, and customer-related costs. The fundamental managerial challenge is consequently to determine not only whether PM should be performed, but also when it should be performed and at what intensity. Recent research has increasingly moved from conventional age-based maintenance policies toward condition-based, dynamic, and integrated maintenance strategies that explicitly consider equipment degradation, production requirements, and product quality [2], [4], [5].

The interdependence between equipment condition and product quality makes the joint consideration of maintenance and quality particularly important. As production equipment deteriorates, the probability of producing nonconforming products may increase, while quality deterioration can itself accelerate equipment degradation in some manufacturing environments. Consequently, maintenance decisions influence both the reliability of the production system and the quantity and quality of its output.

Recent studies have therefore developed integrated production–maintenance–quality models in which inspection, maintenance intervention, production decisions, and quality deterioration are considered simultaneously. For example, Wang et al. [3] developed an integrated quality-control and maintenance model considering quality-dependent failures, while Wei et al. [2] incorporated imperfect maintenance and dynamic inspection into an integrated production, maintenance, and quality-control framework. These developments demonstrate the growing importance of moving beyond maintenance-only models toward integrated operational decision-making.

Another important challenge arises from the multi-product and multi-period nature of practical manufacturing systems. Manufacturing facilities frequently produce several products over a finite planning horizon, while product demand, production requirements, setup costs, processing costs, and the consequences of delayed orders may vary across periods. In such environments, maintenance decisions cannot be determined independently of production planning because a maintenance intervention reduces available production time and may influence the firm's ability to satisfy demand. Conversely, postponing maintenance to increase short-term production capacity may increase the likelihood of failure, defective production, and subsequent production losses. Recent research has accordingly investigated the integration of production planning and maintenance decisions in multi-product and multi-period environments. Davari et al. [4], for instance, developed an integrated simulation-fuzzy framework for PM optimization in multi-product production firms, while a more recent data-driven approach has integrated predictive maintenance with multi-period, multi-product production planning using deep learning and mathematical programming [4].

Demand uncertainty further complicates the coordination of production and maintenance decisions. In real manufacturing environments, demand is rarely constant throughout the planning horizon and may fluctuate according to market conditions and customer requirements. A maintenance policy that is appropriate under low-demand conditions may be inappropriate when demand is high and the cost of lost sales or delayed orders is substantial. Thus, maintenance intensity, inspection frequency, production quantity, inventory levels, and failure-related decisions should ideally be evaluated under alternative demand conditions rather than under a single deterministic demand assumption. Recent studies have explicitly incorporated dynamic customer demand and delivery performance into integrated production, quality, and maintenance models, highlighting the need to coordinate equipment-related decisions with market and service requirements [3].

The nature of equipment failures is another important consideration in integrated production and maintenance planning. In practice, equipment deterioration may manifest itself in different ways. A production system may initially move from an in-control state to an out-of-control state, leading to the production of nonconforming items without immediately stopping the production process. Such a condition may only be identified through inspection. In a more severe failure state, the equipment may become unavailable and production may stop completely. These two failure mechanisms have different operational and economic consequences and therefore may require different maintenance and repair responses. Incorporating multiple failure modes into an integrated optimization model provides a more realistic representation of manufacturing systems than assuming a single failure mechanism. This issue is particularly relevant when inspection decisions, nonconforming production, corrective maintenance, and PM are jointly optimized.

Despite the substantial development of integrated production and maintenance models, several research opportunities remain. First, many existing studies focus on a single product, a single failure mechanism, or a limited maintenance policy, whereas practical manufacturing systems may simultaneously involve multiple products, multiple periods, stochastic failures, quality deterioration, and alternative maintenance levels. Second, although recent studies increasingly incorporate dynamic demand, quality deterioration, and condition-based maintenance, the simultaneous consideration of different failure types, inspection decisions, multiple preventive-maintenance levels, and alternative demand scenarios remain relatively challenging. Third, the computational complexity of such integrated models increases substantially as the number of products, periods, scenarios, maintenance levels, and failure states increases, making the development of efficient solution procedures necessary.

Motivated by these gaps, this study develops an integrated multi-period and multi-product production–maintenance–quality optimization model in which two types of equipment failure are considered. The first failure type represents a transition from an in-control to an out-of-control production state, in which nonconforming products may be generated and the condition of the system can be identified through inspection. The second failure type represents a more severe equipment failure that interrupts production and requires corrective maintenance. The proposed framework further incorporates multiple levels of PM, each associated with different maintenance costs, durations, and effects on equipment age. In addition, alternative demand scenarios are considered to represent different market conditions and their implications for production, inspection, maintenance, failures, and lost orders. The objective is to determine production, inspection, and maintenance decisions that maximize the overall economic performance of the manufacturing system while accounting for the interactions among equipment reliability, product quality, production capacity, and demand.

To solve the resulting mixed-integer nonlinear optimization problem, this study combines an exact mathematical programming approach for smaller problem instances with a step-by-step solution procedure and a genetic algorithm for more complex instances. The proposed approach enables the performance of alternative maintenance and inspection policies to be evaluated and provides managerial insights into the relationship between PM intensity, inspection frequency, production performance, failure occurrence, and profitability. In this respect, the study contributes to the literature by integrating production planning, quality-related consequences of equipment deterioration, inspection, multiple levels of PM, and demand scenarios within a unified multi-period and multi-product framework.

## 2 | Literature Riview

The integration of production planning and maintenance decisions has received considerable attention in the operations management and maintenance literature. The main motivation behind this stream of research is that production and maintenance activities compete for the same limited resources and are therefore strongly interdependent. While increasing production rates can improve short-term revenue, intensive utilization of production equipment may accelerate deterioration and increase the probability of failure. Conversely, PM can improve equipment reliability and reduce the likelihood of unexpected breakdowns, but it requires maintenance costs and production downtime. Consequently, determining production and maintenance decisions independently may result in suboptimal operational performance.

Early studies established the importance of jointly considering production and maintenance decisions. For example, Abdolniur et al. [6] investigated the effect of maintenance policies on just-in-time production systems and demonstrated that maintenance decisions could significantly affect production-system performance. Gharbi and Kenne [7] developed production and preventive-maintenance policies for manufacturing systems with failures and emphasized the interaction between production rates and maintenance decisions. Similarly, Chelbi and Ait-Kadi [8] analyzed a production/inventory system with randomly failing production equipment subject to regular PM. These studies provided important foundations for the subsequent development of integrated production–maintenance models. The original manuscript also builds on this research stream through studies concerning production planning, PM, inventory, and unreliable production systems.

More recent studies have extended this literature by considering more realistic production environments. Davari et al. [4], for example, developed an integrated simulation-fuzzy framework for preventive-maintenance optimization in multi-product production firms. Their study demonstrates that maintenance decisions become substantially more complex when several products and production requirements are considered simultaneously. In another direction, Wang et al. [9] investigated the joint optimization of production scheduling and PM under frequent production switching, highlighting the importance of coordinating maintenance interventions with production schedules.

A common characteristic of these studies is that maintenance is not treated simply as a technical decision concerning equipment replacement or repair. Rather, maintenance is increasingly viewed as an operational decision whose consequences extend to production capacity, scheduling, inventory, customer service, and overall profitability. This perspective provides an important basis for the integrated framework developed in the present study.

Although production–maintenance integration can improve system performance, considering equipment reliability without explicitly accounting for product quality may not adequately represent actual manufacturing systems. Equipment deterioration can increase the probability of producing nonconforming products, particularly when deterioration affects process stability or operating precision. Consequently, production systems may experience two simultaneous consequences of equipment degradation: reduced production availability and deterioration in product quality.

The integration of quality and maintenance decisions has therefore emerged as an important research direction. Bousaleh et al. [10], for instance, investigated joint production, quality, and maintenance control in a two-machine production line subject to operation-dependent and quality-dependent failures. Their study demonstrated the importance of considering quality-related failures when determining production and maintenance policies. Similarly, Beheshti Fakher et al. [11] developed a multi-period and multi-product profit-maximization model integrating production, maintenance, and quality decisions. This study is particularly relevant to the present research because it explicitly combines the three decision areas within a multi-period and multi-product environment. The original article identifies this study as one of the principal foundations for its proposed model.

Recent research has further strengthened this research stream. Lv et al. [12] investigated joint optimization of production, maintenance, and quality control while considering the effect of product-quality variation in a degraded production system. Their results emphasize that degradation should not be modeled solely through its effect on equipment availability because it may simultaneously affect the quality characteristics of manufactured products.

Wang et al. [3] also developed a joint optimization model for quality control and maintenance in a production system with quality-dependent failures. More recently, Wei et al. [2] proposed an integrated production, maintenance, and quality-control framework incorporating imperfect maintenance and dynamic inspection. These studies indicate a clear transition in the literature from traditional maintenance optimization toward integrated decision-making in which equipment condition, production performance, inspection, and product quality are considered simultaneously.

This line of research is directly related to the present study. However, the proposed model differs from many existing models by explicitly incorporating two different failure mechanisms, multiple preventive-maintenance levels, inspection decisions, and alternative demand scenarios within a multi-period and multi-product framework.

Inspection is an important component of manufacturing systems in which equipment deterioration may not immediately result in complete production stoppage. In such systems, a production unit may move from an in-control state to an out-of-control state and continue operating while producing a proportion of nonconforming products. Since this type of deterioration may not be directly observable, inspection provides a mechanism for identifying the system's operational and quality condition.

The relationship between inspection, maintenance, and production performance has been investigated in several studies. Cai et al. [13], for example, jointly considered inspection, maintenance, and spare-parts provisioning and compared alternative inventory policies using simulation and empirical information. Azadeh et al. [14] investigated a joint quality-control and preventive-maintenance strategy and demonstrated the importance of coordinating quality inspection with maintenance decisions.

The present study incorporates this trade-off by considering inspection at different points during the planning horizon and linking inspection outcomes to the operational state of the production system. When an out-of-control condition is detected, nonconforming products are separated from standard products and an appropriate maintenance response is considered. This structure allows the model to capture the economic consequences of delayed detection as well as the cost associated with more frequent inspection.

Traditional preventive-maintenance models often assume that maintenance either completely restores equipment to an as-good-as-new condition or follows a predetermined replacement policy. Such assumptions may not accurately represent many practical manufacturing environments, where maintenance can be performed at different levels depending on the severity of deterioration, available resources, production requirements, and the desired reliability level.

Recent maintenance research increasingly considers imperfect maintenance, in which a maintenance intervention reduces equipment age or deterioration but does not necessarily restore the system completely to its original condition. This approach provides greater flexibility for modeling real manufacturing systems. The original study already adopts this perspective by assigning different costs and durations to different preventive-maintenance levels and by assuming that the maintenance level affects the reduction in the effective age of the production system.

This feature creates an additional optimization problem. A higher maintenance level may reduce the probability of future failures more effectively, but it generally requires more time and expenditure. A lower maintenance level, on the other hand, may preserve production capacity in the short term but provide less protection against future failures. Therefore, the optimal maintenance decision depends on the interaction between maintenance cost, production capacity, failure risk, product quality, and demand.

Another important dimension of the present problem is demand uncertainty. Most real manufacturing systems operate in environments where customer demand varies across time and where the consequences of failing to satisfy demand may differ depending on market conditions. Consequently, production and maintenance decisions based on a single deterministic demand level may not perform well under alternative demand conditions.

Scenario-based approaches provide a useful mechanism for representing these differences. Instead of assuming a single demand level, alternative scenarios can be defined according to different market conditions and associated customer-service requirements. The original model follows precisely this approach by considering different demand scenarios and allowing the required production volume, number of inspections, failures, and lost orders to vary across scenarios.

This scenario-based structure is particularly relevant to the present research because it allows maintenance and production decisions to be evaluated from a broader economic perspective rather than solely on the basis of equipment reliability. In this way, the proposed model connects technical decisions concerning equipment maintenance with market-related consequences such as customer satisfaction, delayed orders, and lost revenue.

Based on the above literature, the present study develops an integrated multi-period and multi-product production, quality, inspection, and preventive-maintenance model. The main distinguishing characteristics of the proposed framework can be summarized as *Table 1*.

**Table 1. Main features of the considered problem.**

| Dimension | Existing Literature | Present Study |
|---|---|---|
| Production planning | ✓ | ✓ |
| PM | ✓ | ✓ |
| Multi-period planning | Some studies | ✓ |
| Multi-product production | Some studies | ✓ |
| Product quality | Some studies | ✓ |
| Inspection decisions | Some studies | ✓ |
| Imperfect PM | Some studies | ✓ |
| Multiple PM levels | Limited | ✓ |
| Two distinct failure types | Limited | ✓ |
| Demand scenarios | Some studies | ✓ |
| Lost/delayed orders | Some studies | ✓ |
| Joint economic optimization | ✓ | ✓ |
| Exact + heuristic solution approaches | Some studies | ✓ |

The principal contribution of the present study therefore lies not merely in adding another preventive-maintenance policy, but in integrating several interdependent operational decisions within a unified framework. Specifically, the model simultaneously determines production, inspection, and maintenance decisions while considering two types of stochastic failures, multiple preventive-maintenance levels, nonconforming production, alternative demand scenarios, and the economic consequences of delayed or lost orders.

This positioning also provides a clearer basis for evaluating the contribution of the proposed model against the most relevant studies in the literature, particularly the integrated production–maintenance–quality model of Beheshti Fakher et al. [11] and more recent integrated quality and maintenance studies such as Lv et al. [12], Wang et al. [3], and Wei et al. [2].

# 3 | Problem Definition and Formulation

This study addresses a production system wherein; a set of different products are produced. This system faces with two types of probable failures. Failure type I is the situation when the system shifts from in-controlled to out-of-control while failure type II is the situation when the system goes out of reach and production is stopped. This model is introduced with a time horizon H and several discrete periods t with a constant length L. At the beginning, the system is assumed to be under control and is producing. In type I failure, the machine is out of control and products of low quality (non-standard) are produced. This kind of failure is only determined by inspection. Type II failure is specified without inspection. In this case system stops and does not produce. *Fig. 1* shows the structure of the system in different cases.

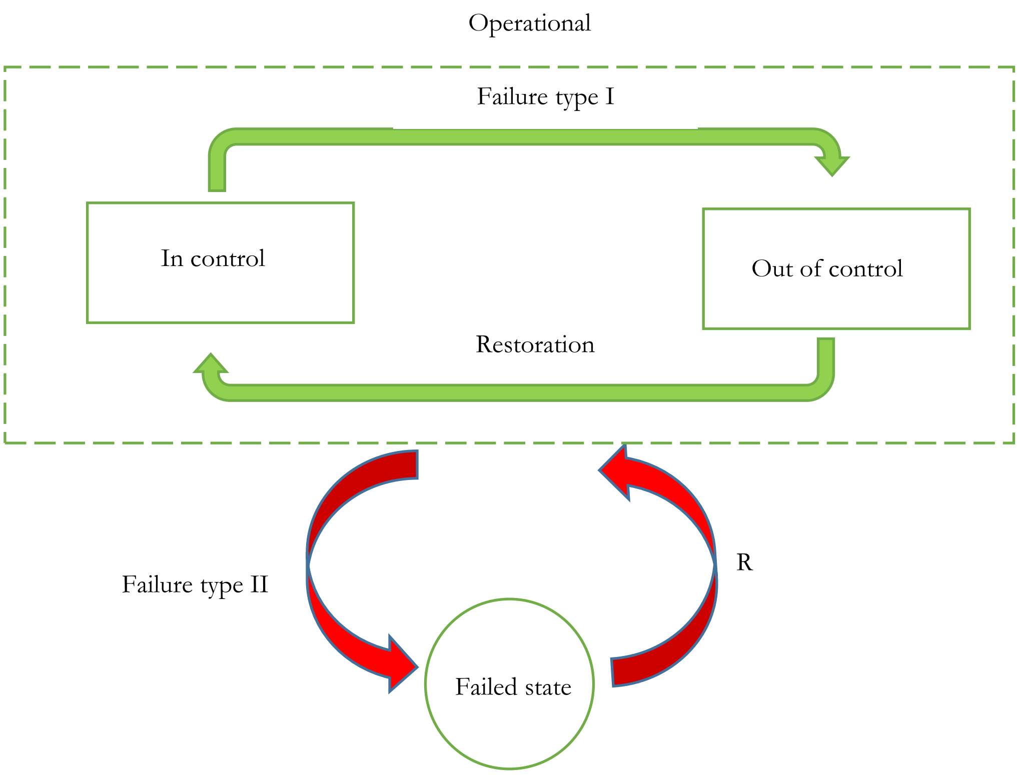


**Fig. 1. States of the production system** [11]**.**

In any period, t the system is inspected and PM carried out, if necessary, which causes machine age reduction. This decrease in age is due to the level of preventive operation. Each time it is determined that the system is out of control, the separation of standard and non-standard products is performed. At the end of each period, a complete system repair is done to zero.

Inspection operations are carried out on manufactured products to check their quality. If the product is detected in low quality, the production is stopped and the condition is announced out of control. Otherwise, the PM applies to the system. The preventive level is limited and these levels have different cost and time.

There are two types of repairs for the model: complete repairs performed at the end of each period and defective repairs performed in the event of Type II failure. Type I and II failures occur with probability, and each one has a defined probability distribution function with specific parameters. *Fig. 2* shows the PM and inspection in different periods.

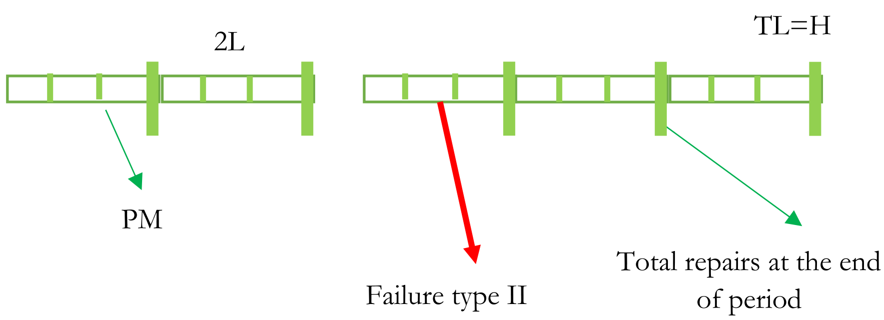


**Fig. 2. PM and inspection in different periods.**

To bring this model closer to real world, different scenarios are considered. Product demand p in each period t is not constant. So that, there are defined scenarios in high, medium and low-level demand to examine the percentage of customer satisfaction in each case. Meanwhile, in addition to the fact that demand is different in each scenario, the volume of products that should be run by the system without an underdeveloped order also varies in different scenarios. For this purpose, the number of inspections and failures that occur in each scenario is different, and the system seeks to maximize the final profit of all these scenarios.

It should be noted that the demand of main customers is only supplied with standard products, and non-standard products are sold at second-party prices in free market and do not have the ability of responding to customers in each period. *Fig. 3* shows an overview of different scenarios based on demand and response rates.

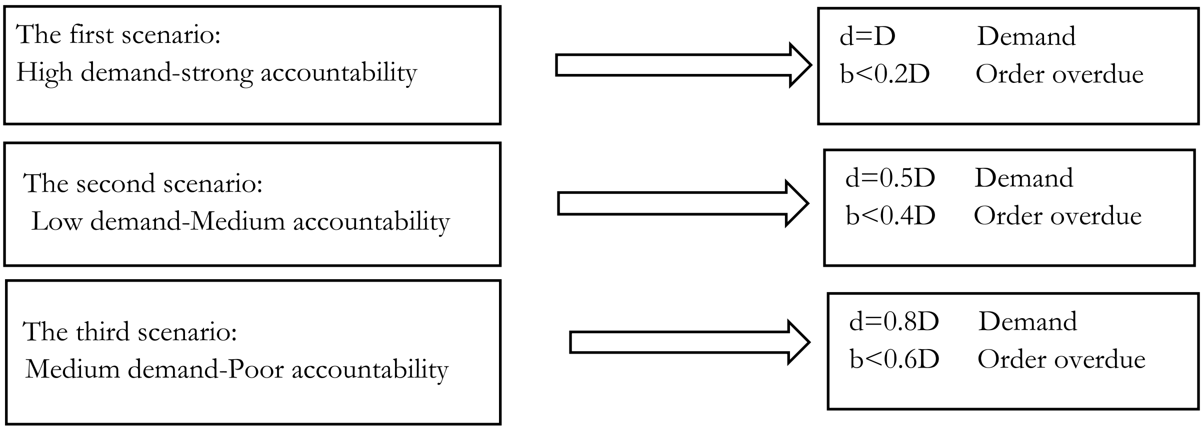


**Fig. 3. Different scenarios to be defined on the model.**

To describe the model, the mathematical model is firstly described with respect to the objective function and the constraints, and then the objective and constraints will be discussed. Parameters and variables of the mathematical model are defined as below:

| **Indices and Parameters** | |
|---|---|
| H | Time horizon |
| L | Period duration |
| T | Number of durations |
| t | Period indicator |
| P | Product number |
| p | Product indicator |
| Q | Number of PM levels |
| q | PM levels indicator |
| k | Indicator of inspection interval and PM |
| CMR | Minimum repair cost |
| TMR | Minimum repair time |
| CCR | Full repair cost |
| $\beta$ | Once inspection cost |
| $\eta$ | PM factor |
| $\xi$ | Resettlement cost |

| | |
|---|---|
| $F(u)$ | Cumulative Distribution Function of shift time (Type I Failure) |
| $f(u)$ | Distribution Function of shift time (Type I Failure) |
| $r(u)$ | Risk function of Type I failure |
| $G(u)$ | Cumulative Distribution Function of type II failure time |
| $g(u)$ | Distribution Function of type II failure time |
| $\zeta(u)$ | Risk function of Type II failure |
| $\lambda,\varphi$ | Weibull Distribution Parameters of Failure Type I |
| $\theta,\rho$ | Weibull Distribution Parameters of Failure Type II |
| $I_t^k$ | The length of inspection interval k in period t |
| $d_{t,s}^p$ | Demand of product p in period t for scenario s |
| $\pi_t^p$ | Production process cost for product p in period t |
| $b_t^p$ | Delayed order cost for product p in period t |
| $h_t^p$ | Maintenance cost for product p in period t |
| $s_t^p$ | System startup cost for product p in period t |
| $\delta^p$ | Separation unit cost for product p |
| $g^p$ | Production rate for product p |
| $\alpha^p$ | Non-standard product rate in an out-of-control system |
| $PC_t^p$ | Price of standard product p during period t |
| $PN_t^p$ | Price of non-standard product p during period t |
| $PM(q)$ | PM cost in level q |
| $TPM(q)$ | PM time in level q |
| **Decision Variables** | |
| $NF_{t,s}^k$ | The number of Type II failures in interval k of period t in scenario s |
| $APT_{t,s}$ | Production time t available in scenario s |
| $PS_{t,s}^k$ | shift probability in interval k of period t in scenario s |
| $QC_{t,s}$ | The cost of separating products in period t and scenario s |
| $w_{t,s}^k$ | System starting age in interval k of period t in scenario s |
| $y_{t,s}^k$ | System ending age in interval k of period t in scenario s |
| $XN_{t,s}^p$ | Number of non-standard product p in period t and scenario s |
| $XC_{t,s}^p$ | Number of standard product p in period t and scenario s |
| $x_{t,s}^p$ | Product p volume in period t and scenario s |
| $B_{t,s}^p$ | Product p delayed order in period t and scenario s |
| $IC_{t,s}^p$ | Standard product p inventory in period t and scenario s |
| $IN_{t,s}^p$ | Standard product p inventory in period t and scenario s |

| | |
|---|---|
| $S^{p}_{t,s}$ | Binary decision variable for system startup for generating product p in period t |
| $m_{t,s}$ | Number of PM activities in period t and scenario s |
| $M^{k}_{t,s}$ | PM level in interval k of period t in scenario s |
| $SC^{p}_{t,s}$ | Number of standard product p sold in period t and scenario s |
| $SN^{p}_{t,s}$ | Number of non-standard product p sold in period t and scenario s |

# 4 | Objective Function

The considered objective Function is a profit maximization function that derives from diversity of sales revenue and operating costs. Operating costs include: production costs, maintenance costs, startup costs, order lost cost, inspection costs, PM costs, redevelopment costs, the cost of separating standard and non-standard products and failure type II. Sale of products include standard and non-standard products that happens due to machine age increase. The objective function is as follows:

$$\begin{aligned}
&PP_s\Big(\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{p=1}^{P}(PC^{p}_{t}SC^{p}_{t,s}+PN^{p}_{t}SN^{p}_{t,s})\\
&-\Big[\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{p=1}^{P}(\pi^{p}_{t}x^{p}_{t,s}+h^{p}_{t}(IC^{p}_{t,s}+IN^{p}_{t,s})+s^{p}_{t}S^{p}_{t,s}+b^{p}_{t}B^{p}_{t,s})\Big]\\
&-\beta\sum_{t=1}^{T}(m_{t,s}+1)-\sum_{t=1}^{T}\sum_{k=1}^{m_{t,s}}CPM(M^{k}_{t,s})-\xi\sum_{t=1}^{T}\sum_{k=1}^{m_{t,s}+1}\Big(\int_{w^{k}_{t,s}}^{y^{k}_{t,s}}(y^{k}_{t,s}-u)f(u\mid w^{k}_{t,s})du\Big)PS^{k}_{t,s}\\
&-\sum_{t=1}^{T}QC_{t,s}-(T\times CCR)-CMR\sum_{t=1}^{T}\sum_{k=1}^{m_{t,s}+1}NF^{k}_{t,s}\Big).
\end{aligned} \qquad \textbf{(Obj.)}$$

**Constraints**

$$IC^{p}_{t,s}=IC^{p}_{t-1,s}-SC^{p}_{t,s}+x^{p}_{t,s}-XN^{p}_{t,s},\quad \forall t,p,s, \qquad (1)$$

$$IN^{p}_{t,s}=IN^{p}_{t-1,s}-SN^{p}_{t,s}+XN^{p}_{t,s},\quad \forall t,p,s, \qquad (2)$$

$$B^{p}_{t,s}=B^{p}_{t-1,s}+d^{p}_{t,s}-SC^{p}_{t,s},\quad \forall t,p,s, \qquad (3)$$

$$x^{p}_{t,s}\le L.g_p.S^{p}_{t,s},\quad \forall t,p,s, \qquad (4)$$

$$\sum_{p=1}^{P}\frac{x^{p}_{t,s}}{g_p}\le APT_{t,s},\quad \forall t,s, \qquad (5)$$

$$w^{1}_{t,s}=0,\quad \forall t,s, \qquad (6)$$

$$w^{k+1}_{t,s}=y^{k}_{t,s}(1-(M^{k}_{t,s}/CPM('4'))\eta^{k-1}),\quad \forall t,s, \qquad (7)$$

$$y^{k}_{t,s}=w^{k}_{t,s}+I^{k}_{t,s}-TPM(M^{k}_{t,s})-NF^{k}_{t,s}TMR,\quad \forall t,k,s, \qquad (8)$$

$$NF^{k}_{t,s}=\int_{w^{k}_{t,s}}^{y^{k}_{t,s}}\zeta(u)du,\quad \forall t,k,s, \qquad (9)$$

$$APT_{t,s}=\sum_{k=1}^{m_t+1}(y^{k}_{t,s}-w^{k}_{t,s}),\quad \forall t,k,s, \qquad (10)$$

$$PS^{k}_{t,s}=\frac{F(y^{k}_{t,s})-F(w^{k}_{t,s})}{1-F(w^{k}_{t,s})},\quad \forall t,k,s, \qquad (11)$$

$$XN^{p}_{t,s} = x^{p}_{t,s} \frac{\alpha^{p}}{APT_{t,s}} \sum_{k=1}^{m_{t,s}} \int_{w^{k}_{t,s}}^{y^{k}_{t,s}} PS^{k}_{t,s}(y^{k}_{t,s} - u) f(u \mid w^{k}_{t,s}) du, \quad \forall t,k,p,s, \tag{12}$$

$$QC_{t,s} = \sum_{t=1}^{T} \frac{\delta^{p} x^{p}_{t,s}}{APT_{t,s}} \sum_{k=1}^{m_t} PS^{k}_{t,s}(y^{k}_{t,s} - w^{k}_{t,s}), \quad \forall t,s, \tag{13}$$

$$B^{p}_{t,s} \leq qp_s \times d^{p}_{t,s}, \quad \forall t,p,s, \tag{14}$$

$$IC^{p}_{0,s} = 0, IN^{p}_{0,s} = 0, B^{p}_{0,s} = 0, \quad \forall p,s, \tag{15}$$

$$M^{k}_{t,s} \in \{1,2,...,Q\}, \quad \forall t,k,s, \tag{16}$$

$$m_{t,s} \geq 0, \quad \forall t,s, \tag{17}$$

$$S^{p}_{t,s} \in \{0,1\}, \quad \forall t,p,s, \tag{18}$$

$$x^{p}_{t,s} \geq 0, SC^{p}_{t,s} \geq 0, SN^{p}_{t,s} \geq 0, IC^{p}_{t,s} \geq 0, IN^{p}_{t,s} \geq 0, B^{p}_{t,s} \geq 0, \quad \forall t,p,s. \tag{19}$$

*Constraints (1)* and *(2)* shows the standard and non-standard inventory of the product p during period t in scenario s resulted of previous period inventory plus products of this period minus the sale of this period. *Constraints (4)* and *(5)* indicate the volume of product p generation during period t based on system availability time and production rate of product p. *Constraint (6)*, determines that the machine age at the beginning of each period t is zero. *Constraints (7)* and *(8)* compute the age-related relationships at the beginning and end of each period after PM in each scenario s. *Constraint (9)* shows the number of type II failure in interval k in period t in scenario s. *Constraint (10)* calculates system's availability time to produce products. *Constraint (11)* shows the probability of a shift in interval k of period t. *Constraint (12)* shows the number of nonstandard production based on system age and availability in each scenario. *Constraint (13)* calculates the cost of separating standard and non-standard products according to system age and availability and production volume. *Constraint (14)* shows maximum allowed sale in each scenario. *Constraints (15)-(19)* express the type of variables.

In addition, the probability distribution considered for Type I and Type II failure is Weibull distribution.

Proposed mathematical model and existing variables, introduces a Mixed-integer nonlinear Programming model. This model in small dimensions is solved in GAM software with solvent MNLP and the results are presented. Given the complexity of the model, the exact solution in GAMS provides a proper upper limit for solving in smaller dimensions.

# 5 | Solving Procedure

To solve the problem at hand, two approaches are proposed.

I. A step-by-step approach which lets to discuss objective function and obtain the amount of profit by setting different values for number and type of PM.

Due to complexity of the model and probable states available for solving the model, a method is proposed to reduce the complexity related to nonlinearity of the model. This method consists of four basic steps, as shown in *Fig. 4*.

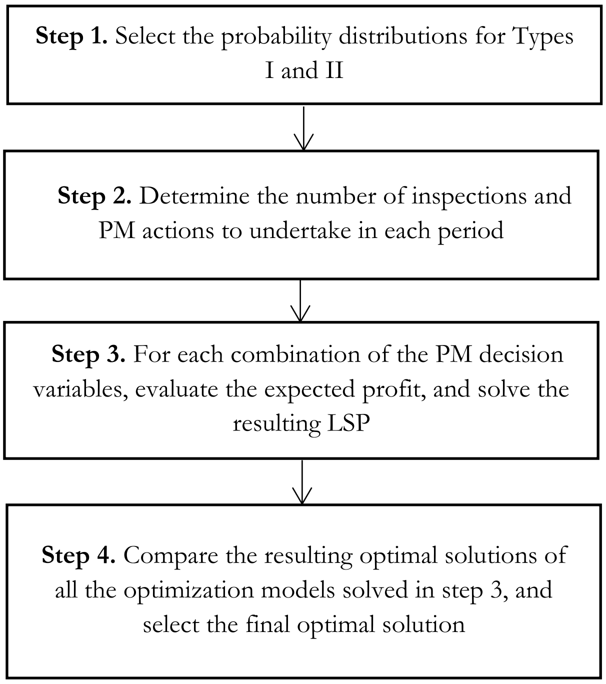


**Fig. 4. Implementation levels for step-by-step approach.**

**Step 1.** probability distributions of Type I and II failures are indicated. In this paper, the probability of failures follows Weibull distribution, which increases the probability of failure and producing non-standard parts as the functionality and age of the machine increase. The probability density function of type I failure is as *Eqs. (20)-(25).*

$$f(u) = \lambda\varphi u^{\varphi-1}e^{-\lambda u^{\varphi}}, u > 0, \varphi \geq 1, \lambda > 0. \tag{20}$$

While λ and φ are Weibull parameters.

Its corresponding cumulative distribution function is:

$$F(t) = 1 - e^{-\lambda u^{\varphi}}, u > 0, \varphi \geq 1, \lambda > 0. \tag{21}$$

Its hazard rate is equal to:

$$r(u) = \lambda\varphi u^{\varphi-1}, u > 0, \varphi \geq 1, \lambda > 0. \tag{22}$$

The probability density function of Type II failures is also Weibull with parameters θ and ϱ as follows:

$$g(u) = \theta\rho u^{\rho-1}e^{-\theta u^{\rho}}, u > 0, \rho \geq 1, \theta > 0. \tag{23}$$

Its corresponding cumulative distribution function is:

$$G(u) = 1 - e^{-\theta u^{\rho}}, u > 0, \rho \geq 1, \theta > 0. \tag{24}$$

Its hazard rate is equal to:

$$\zeta(u) = \theta\rho u^{\rho-1}, u > 0, \rho \geq 1, \theta > 0. \tag{25}$$

**Step 2.** After determining the type of failure distributions, the number of inspections required in each period and the type of PM activity are specified. The specified inspections are carried out during the period and are performed in given intervals to simplify the problem. The length of time interval required for inspection is obtained from the following formula:

$$I_t^k = \frac{L}{m_t + 1}, t = 1, \dots, T, k = 1, \dots, m_t. \tag{26}$$

**Step 3.** After determining failure rates, in this step equations 6 to 9 are solved for each PM level. From the solution of these equations, the following result is obtained which leads to an estimation of the system age in the beginning and the end in each inspection period.

$$\left[\theta \times (y_t^k)^{\rho} \times \mathrm{TMR}\right] + y_t^k = w_t^k + l_t^k - \left(\mathrm{TPM} \times M_t^k\right) + \left[\theta \times (w_t^k)^{\rho} \times \mathrm{TMR}\right]. \quad \textbf{(27)}$$

At this stage, the start and end age and failure distributions can be obtained for different types of PM modes. After placing the specified values, decision variables of inventory amount, shortage amount, production amount of standard and non-standard products, and sales amount of each product should be calculated. The nonlinear model transforms into a simple linear model which can be solved with any software possible.

**Step 4.** Finally, the results from different PM modes are compared to determine the best optimal mode for the system.

II. The genetic metaheuristic method, in which different modes of PM, inspection, and production volume are used as elements of the population.

Genetic Algorithm as a metaheuristic implementation needs to decide how to represent solutions in an efficient way to the searching space [15], [16]. Representation should be easy to decode and calculated to reduce the run time of the algorithm. In the considered problem, the first part of deriving the answer, includes variables related to production volume in each period, the second part includes startup variables and the third part includes types of PM in each inspection period. *Fig 5* shows initial answer determining.

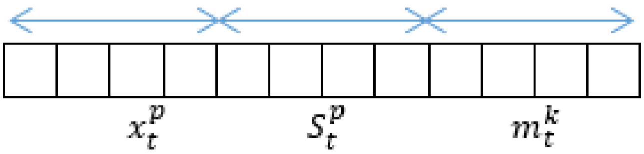


**Fig. 5. Initial answer determining.**

After placing parameters and constructing initial response, genetic parameters are determined. Final parameters are chosen based on the problem structure and several consecutive solving problems.

The GA is a population-based metaheuristic, so it needs an initial population of individuals to start. This population can either be produced randomly or based on some other algorithms. In this study, each chromosome of the initial population is generated randomly [16], [17].

After generating the initial population and new generations, the value of the objective function is calculated and considered as fitness value of each solution ($\mathrm{fv}(\mathrm{x})$). In this way, the solutions are sorted according to the values of the value of the objective.

The parent selection method determines how to choose the chromosomes from the current population for the crossing procedure. In this study, the parents are selected based on the roulette-wheel rule. After calculation, the fitness value ($\mathrm{fv}(\mathrm{x})$) for each solution the operation of the roulette-wheel is done to select the parents.

In general, the crossover operator is done to generate a new chromosome (offspring) with more fitness. After that, the mutation operator is applied to maintain the diversity of the population in the successive generations and to more exploit the solution space runs. Some different methods were tested for the crossover, and finally, two-point crossover (2PX) was recognized well than the others. Here, three different operators: including the Swap operator, Reversion operator, and Insertion operator, are applied randomly in each iteration to enhance the performance of the proposed algorithm.

The new generation $P_{t+1}$ is selected from the parents and offspring but not just based on the fitness values. Rather than, in each generation of the proposed GA, a new population is generated by replacing the second 50% of the current population with the first 50% of the new population (offspring). Based on this procedure, there is no guarantee that the inserted best solutions are always better than the worst solutions in the current population. This approach makes the algorithm towards searching the various regions and avoids form a premature convergence during the progress of the algorithm. The flow chart of the proposed GA is shown in *Fig. 6*.

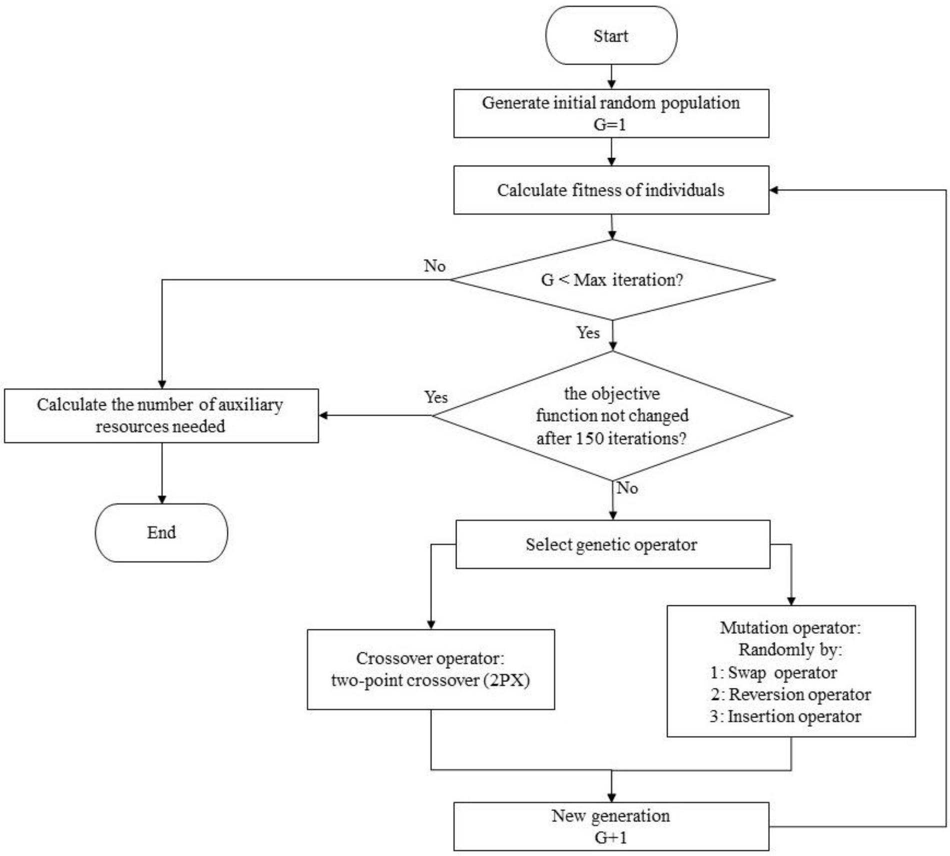


**Fig. 6. Flowchart of the proposed genetic algorithm.**

# 6 | Results and Discussions

In this paper, initial population is 10, mutation size is 2, and the number of elements for crossover is 18. Stop conditions are investigated in three modes: maximum iteration equals 100, earning a negative profit and having 20 consecutive moves without improvement.

After each iteration, the profit from each population is computed. These populations are arranged based on performance improvement and 3 populations with the least profit are replaced with mutation and crossover operators. For the crossover operator, the first two populations with best profit improvement are selected and a number of their elements moved randomly. This operator produces two children which are placed in the eighth and ninths population position and creates the third child mutation operator. This operator randomly modifies a number of members and tries to create a different child. This child will replace the tenth population.

The best answer for PM types in the three inspection periods is (1 1 4,4 1 3,4 1 4) with a profit function of 4.9275. If production quantity in each period is kept constant for each product (4 4 4.4 4 4.4 4 4) has the worst profit function since no PM occurs and the number of failures and the number of non-standard production increases. *Fig. 7* presents a sample answer of Genetic algorithm:

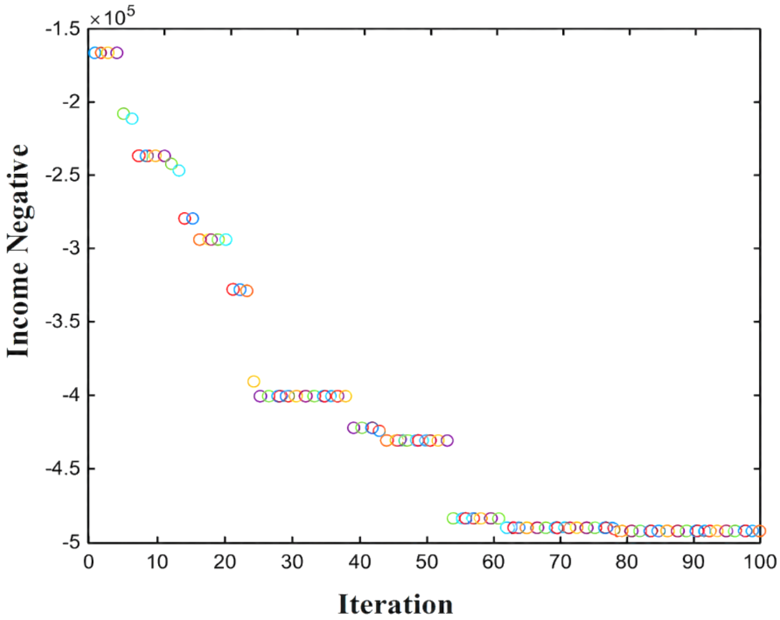


**Fig. 7. Sample answer for genetic algorithm for inspiration third approach.**

The next discussion, which is very important, is the number of inspections required in the system. This number of inspections is investigated according to cost amounts for each inspection, the PM of each period and the costs of failures. According to parameters specified in the model, profit increases as the number of inspections increase. Therefore, according to site exist in industry, the number of necessary inspections is selected. Input parameters and production data are presented in *Tables 2* and *3*: Beheshti Fakher et al. [11].

**Table2. Input parameters.**

| Parameter | Notation | Respective Value |
|---|---|---|
| Cost of PM level q ($) | CPM(q), q=1,2,3,4 | 5000,1000,200,0 |
| Time of PM level q (month) | TPM(q) | 0.05,0.003,0.001,0 |
| Production rate of product p (items/month) | $g^p$ , p=1,2,3 | 450,400,350 |
| Nonconformity rate of product p | $\alpha^p$ | 0.7,0.7,0.7 |
| Cost of separating one product p ($) | $\delta^p$ | 4,5,6 |
| Cost of minimal repair ($) | CMR | 500 |
| Time of minimal repair (month) | TMR | 0.02 |
| Inspection cost ($) | $\beta$ | 40 |
| PM Imperfectness factor | $\eta$ | 0,9 |
| Restoration cost parameter ($/month) | $\xi$ | 3000 |
| Cost of a complete repair ($) | CCR | 6000 |

**Table 3. Data of products.**

| Product | | Demand (Items) | | | Processing Cost ($) | | | Backorder Cost ($) | | | Holding Cost ($) | | | Set-up Cost ($) | | | Price of Conforming ($) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| Period | 1 | 45 | 50 | 100 | 30 | 50 | 70 | 110 | 130 | 180 | 3 | 5,5 | 2,2 | 500 | 800 | 450 | 170 | 320 | 65 |
| | 2 | 30 | 40 | 150 | 26 | 47 | 74 | 110 | 130 | 170 | 2,5 | 6,1 | 2,5 | 550 | 780 | 420 | 150 | 300 | 70 |
| | 3 | 60 | 70 | 50 | 33 | 49 | 68 | 120 | 130 | 170 | 3,2 | 6,5 | 2,4 | 530 | 830 | 400 | 180 | 340 | 68 |

Using *Tables 2* and *3* values, *Fig. 7* shows profit function according to number of inspections. It is observed that the highest amount of profit is obtained when the number of inspections is between 16 and 18.

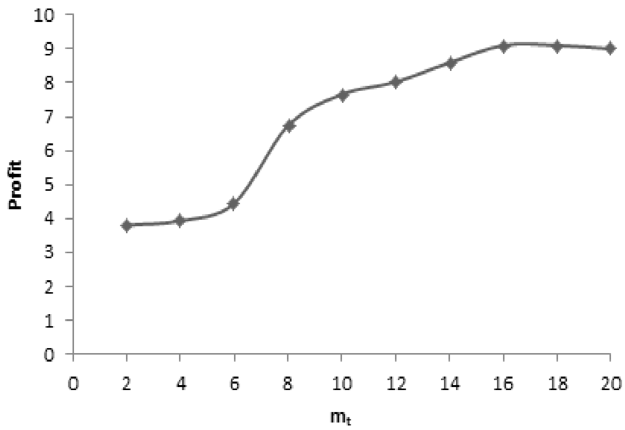


**Fig. 7. Profit function according to number of inspections.**

# 8 | Conclusions

In this paper, we proposed a profit maximizing model which is integrated with quality, production and maintenance decisions. Despite lots of existing models, two types of failures I and II are considered for system's failure probability in this model. Therefore, the production involves two types of standard and non-standard products and inspection factor is used to identify non-standard products manufactured in type I failure. In the case of type II failure, the system would stop and there would be no production. In addition, two types of repairs are applied on system. One is the complete repair at the end of each period and the other is incomplete repair performed as failure Type II occurs. These two types of failure are probable and therefore, statistical distributions are used in this model. Another characteristic of this research which is less likely to be found in other studies is to consider different scenarios. Demand and response rates vary in each period due to different situations in real world, so three scenarios are considered. As this problem is encountered in NP-hard issues, a metaheuristic method is used to solve it. The method used is genetic and software used is GAMS.

Given the highest profit (objective function) the best response is obtained in three inspection periods. On the other hand, the worst case that gives the lowest profit is specified, and this lowest profit is due to the fact that no PM occurs which increases the number of failures and non-standard products manufactured.

Another important issue in this study is determination of inspections number needed to maximize profits, as inspection and PM requires cost. Instead, carrying out inspections and PM activities will reduce or even eliminate non-standard products. The results show that according to the parameters specified in the paper's model, the increase in the number of inspections increases the profit. Therefore, according to the site of industry, the number of necessary inspections is selected.

For developing the topic used in this paper, several studies can be done, including:

I. Extending proposed model in terms of several machines.

II. Using different metaheuristic methods, comparing results and finding the best method in solving this problem.

III. Defining the objective function to minimize costs and consequently the constraints will be changed.

IV. Determining optimal buffer level as one of problem variables due to the difference in demand and response rate in different scenarios.


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## Data Availability

The data used in this study are available from the corresponding author upon reasonable request.

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